

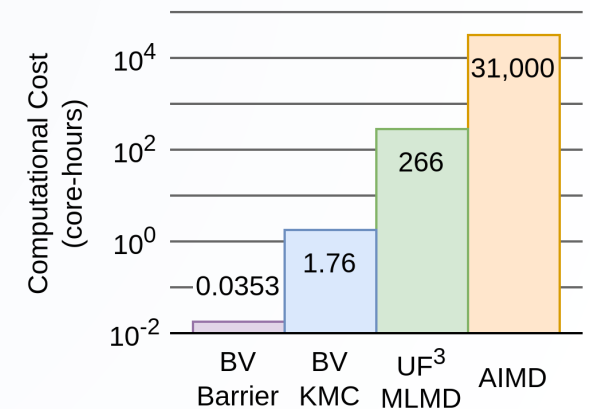
Hierarchical screening for Li-based solid electrolytes using fast, interpretable machine-learned potentials

Stephen R. Xie¹, Shreyas J. Honrao¹, John W. Lawson²

¹ KBR Inc, NASA Ames Research Center

² NASA Ames Research Center, Moffett Field, CA, USA

- Pipeline for high-throughput ionic conductivity predictions
- Validating bond valence methods for screening solid-state electrolytes
- Automatic fitting of ultra-fast, interpretable machine-learning force fields

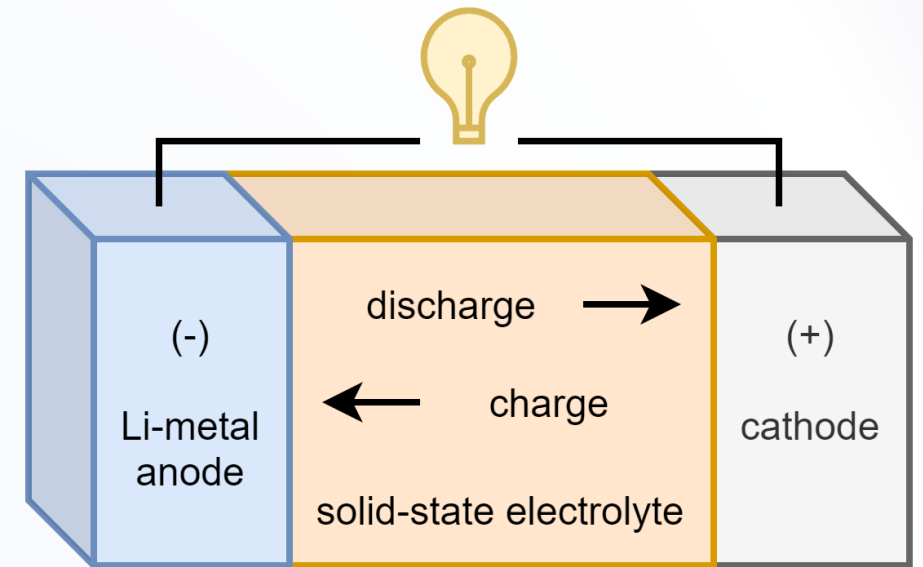


Background

- All Solid-State Batteries with Li anode
 - Safety
 - Energy density
 - Charging rate
- Solid-state Electrolyte
 - High ionic conductivity
 - Low electronic conductivity
 - Good electrochemical stability
 - Abundance, low-cost, manufacturability, etc.

Honrao, S.J., Yang, X., Radhakrishnan, B. *et al.* Discovery of novel Li SSE and anode coatings using interpretable machine learning and high-throughput multi-property screening. *Sci Rep* **11**, 16484 (2021).

<https://doi.org/10.1038/s41598-021-94275-5>



nasa.gov

Computational Screening

- Honrao et al. 2021
 - Generated database of battery-related material properties
 - Screening
 - Thermodynamic stability
 - Electrochemical stability
 - Band gap
 - Trained interpretable machine learning models to predict bond valence (BV) migration barriers, oxidation and reduction potentials
- Migration barrier is a proxy for ionic conductivity
- Goal: construct framework to efficiently compute ionic conductivity for screening

Type	Property	Symbol	Units
Stability	Energy above convex hull	E_{hull}	eV/atom
	Oxidation potential	V_{oxi}	V
	Reduction potential	V_{red}	V
Device	Band gap	E_{g}	eV
	Shear modulus	G	GPa
Transport	Vacancy formation energy	E_{V}	eV
	Migration barrier	E_{M}	eV
	Ionic conductivity	$\sigma_{300\text{K}}$	mS/cm

Ionic Conductivity

- Mean squared displacement (MSD)

- For Li, should grow linearly with time in ionic conductors

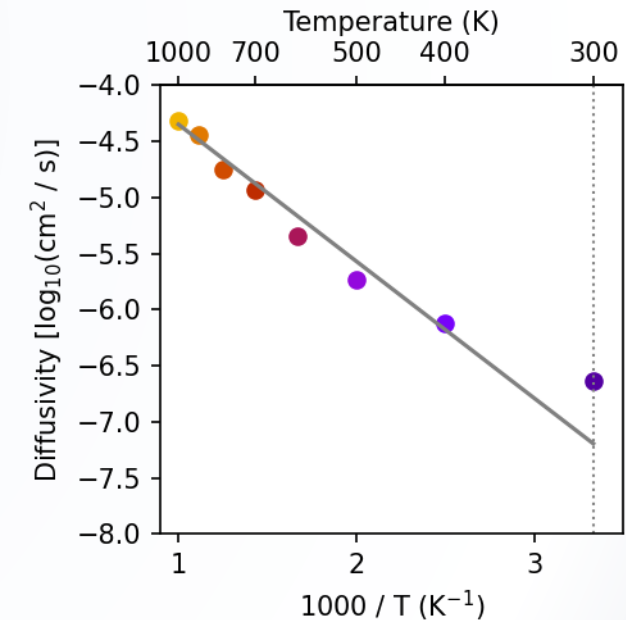
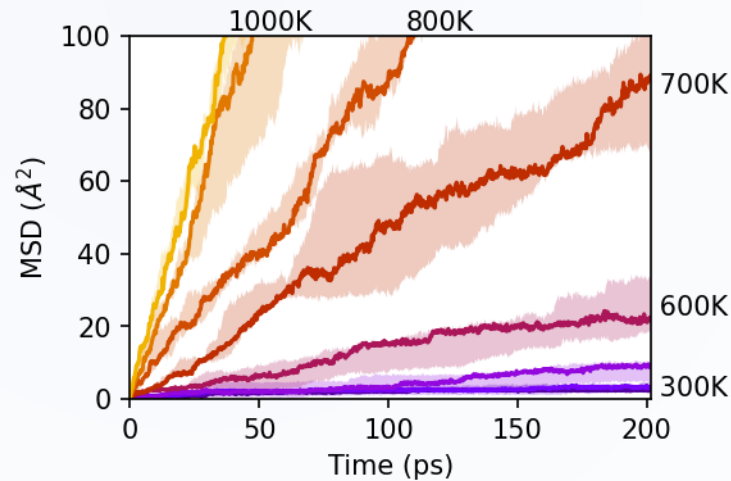
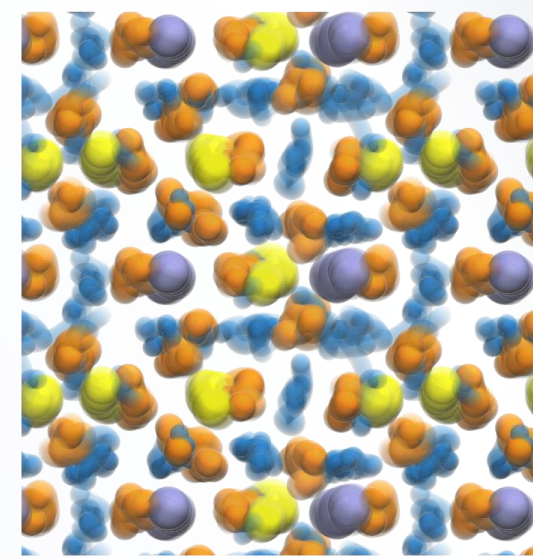
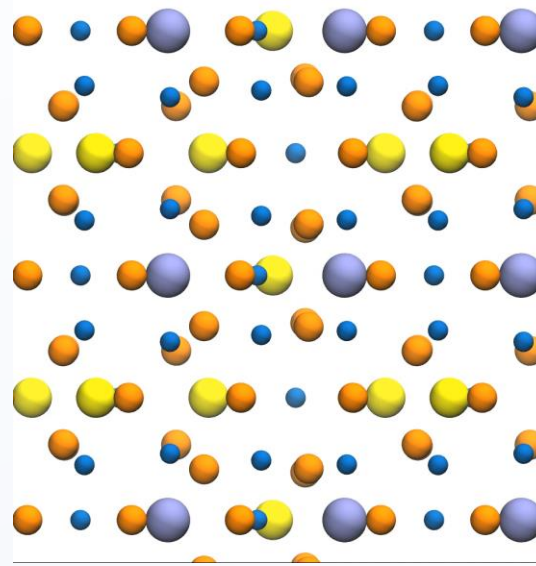
- $$\text{MSD} = \left\langle \left[\overrightarrow{r(t)} - \overrightarrow{r(0)} \right]^2 \right\rangle$$

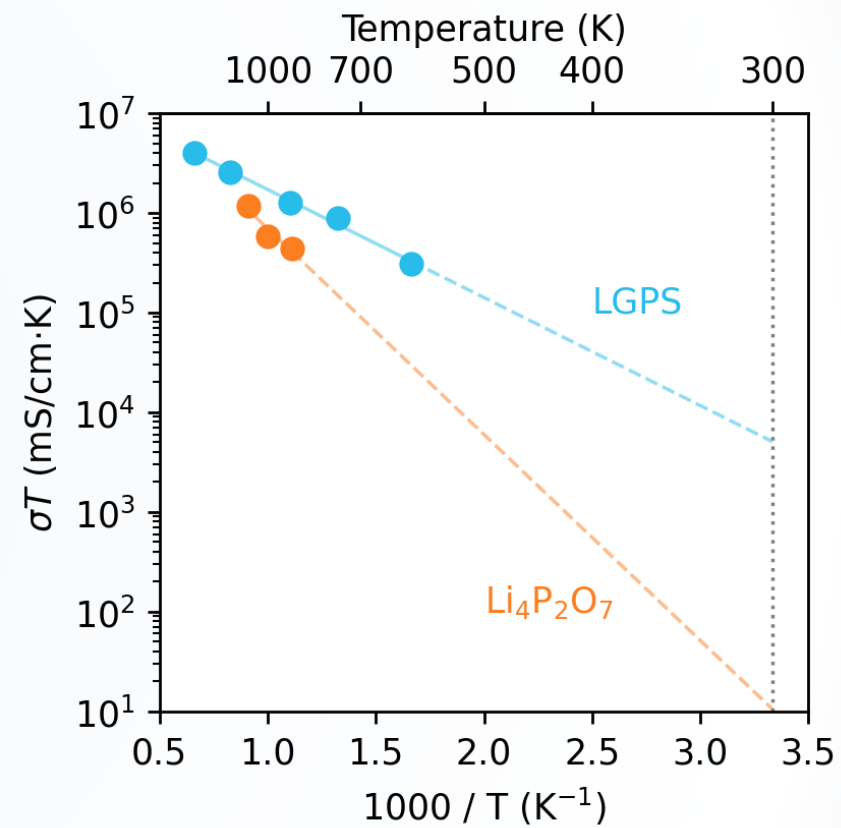
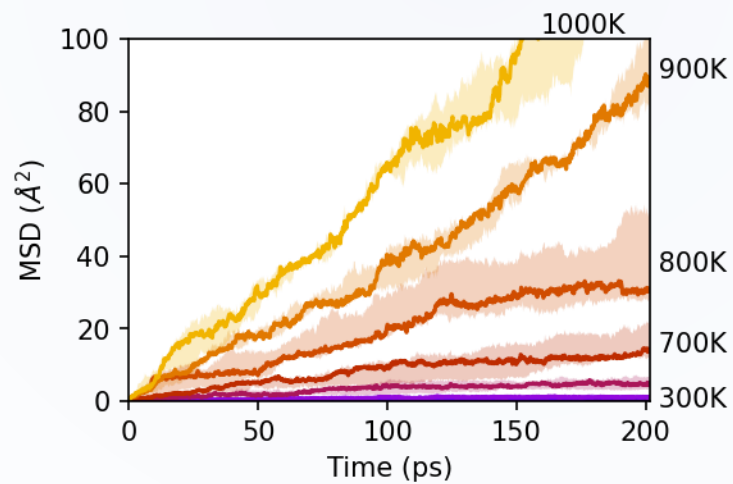
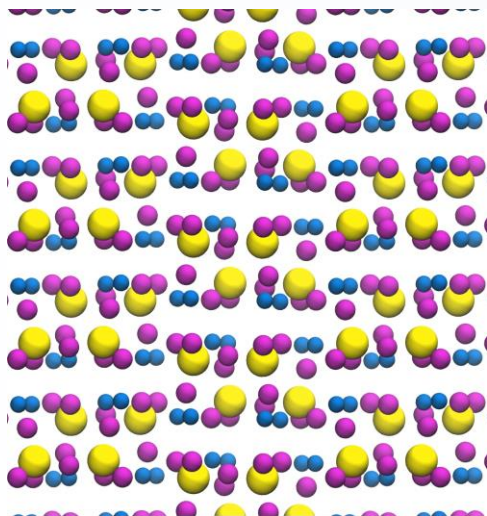
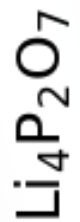
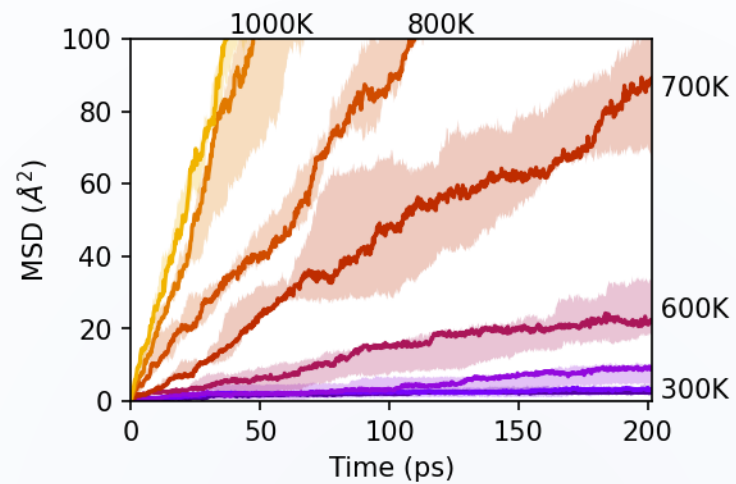
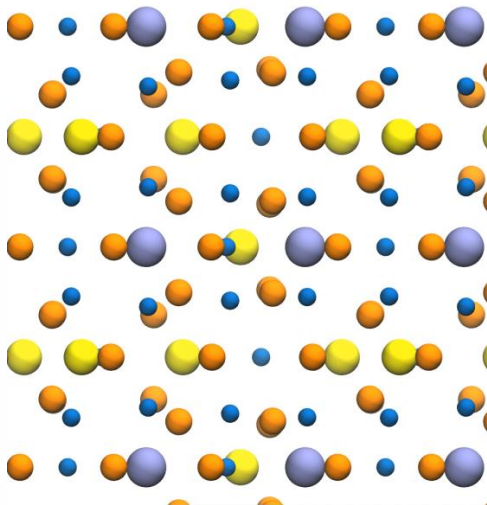
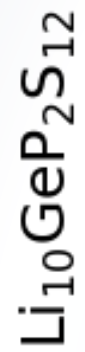
- Diffusion coefficient

- $$D = \frac{1}{2d} \lim_{t \rightarrow \infty} \left\langle \left[\overrightarrow{r(t)} - \overrightarrow{r(0)} \right]^2 \right\rangle$$

- For 3D system (d=3), fit from one-sixth of slope of MSD vs. time

- Ionic Conductivity:
$$\sigma = \frac{\eta q^2}{k_B T} D$$





High-throughput searches

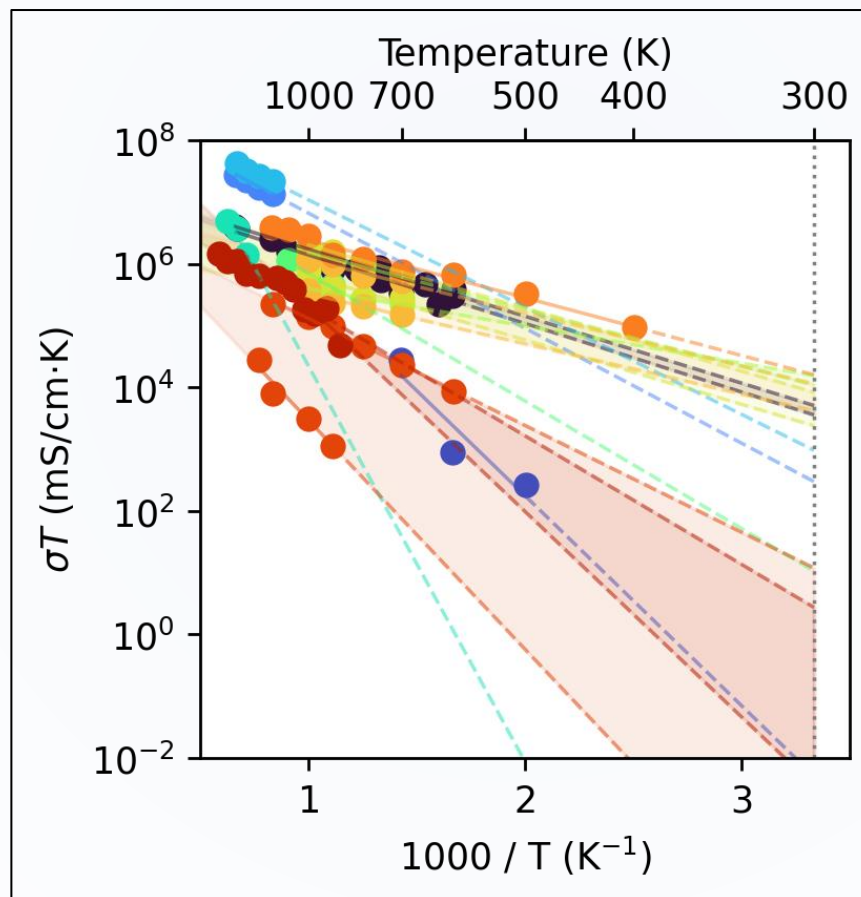
	Data source	Number of candidates			Number of Transport calculations		
		Initial	Before transport	Final	Pathway Analysis	MD	First Principles
Honrao et al. 2021	Materials Project	15466	15466	~250	15446	-	-
Sendek et al. 2017	Materials Project	12831	317	21	-	-	-
Sendek et al. 2019	Materials Project	12831	317	12	-	-	41 (AIMD)
Muy et al. 2019	Materials Project	~14,000	~14,000	18	-	-	~1000 Phonon Band centers
Kahle et al. 2020	ICSD and COD	7472	1016	47	-	916	132 (AIMD)
Wang et al. 2020	Materials Project	1545	234	5	-	19	19 (AIMD at 1000 K)
Wang et al. 2021	Materials Project	1545	274	25	-	36	274 (AIMD at 1000 K)
Xiao et al. 2019	ICSD	104082	184	6	-	-	7 NEB barriers
Xiao et al. 2021	ICSD	6242	53	7	-	-	12 (AIMD at 1000K), 10 (AIMD)
Jun et al. 2022	Materials Project	8572	122	22	-	-	56 (AIMD at 1000K), 22 (AIMD)
Sundberg et al. 2022	Materials Project	9747	3174	12	-	-	6797 IDDP, 299 NEB barriers
Zhao et al. 2022	Materials Project	16205	87	2	87	-	9 (AIMD)

Validation set for ionic conductivity

- Existing ab initio molecular dynamics (AIMD) studies
- Diversity in composition, structure, and ionic conductivity

AIMD

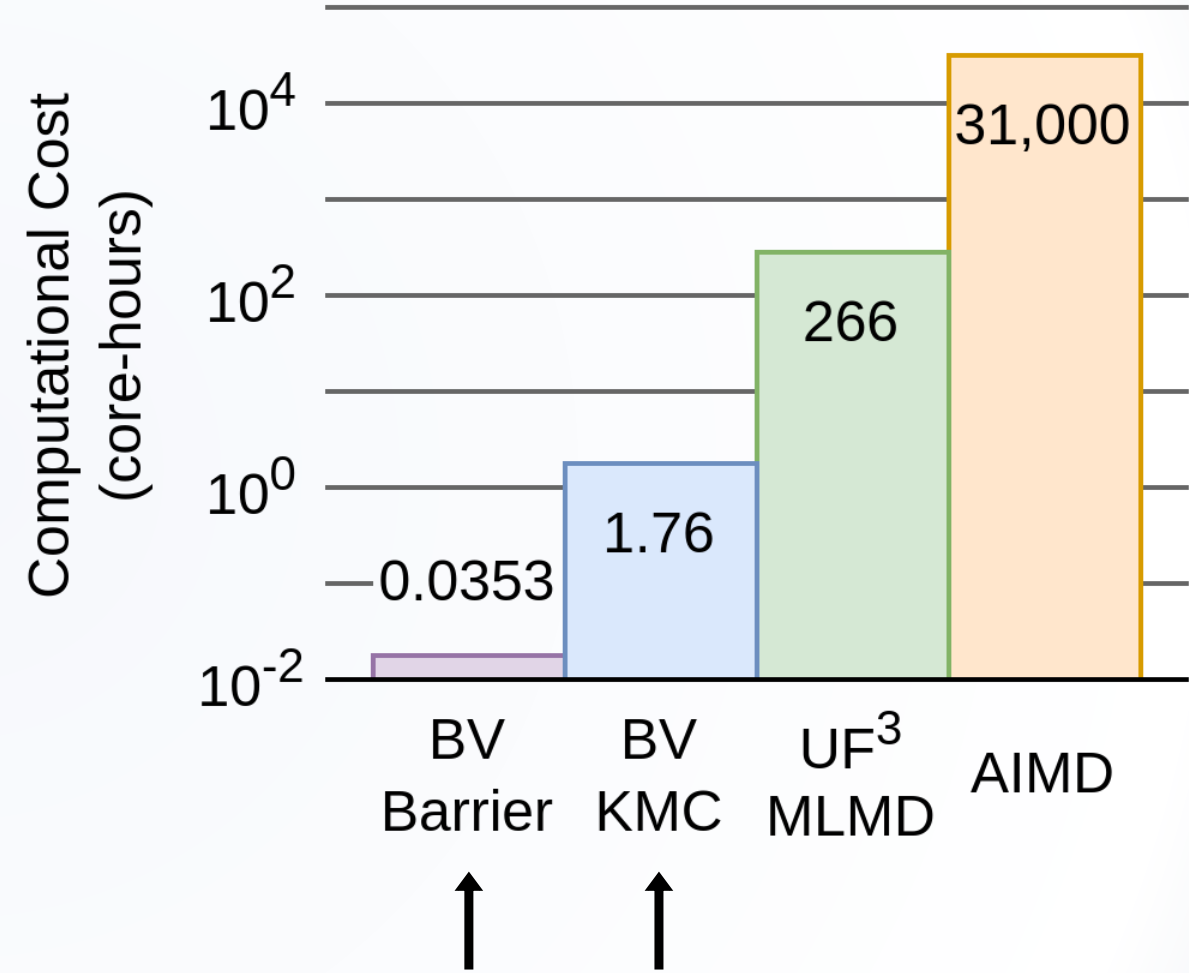
- PBE-DFT
- Large supercell
- Gamma-point only
- 100-500 picoseconds for diffusivity convergence
- Repeat for multiple temperatures to fit Arrhenius relationship



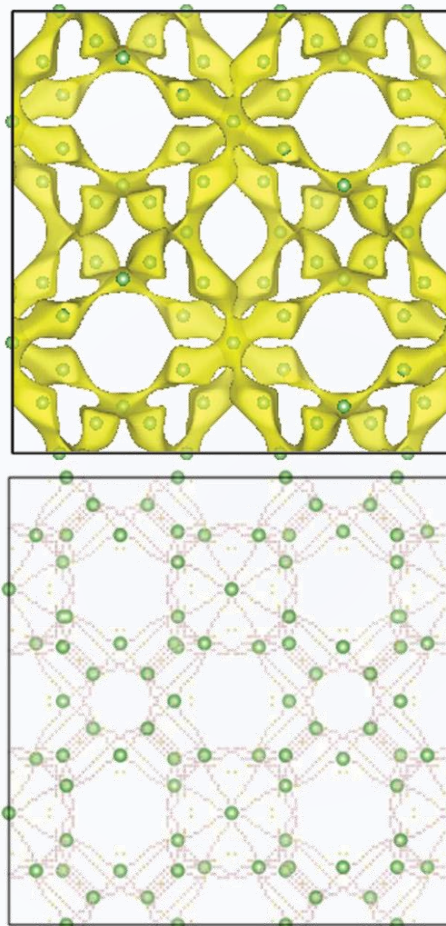
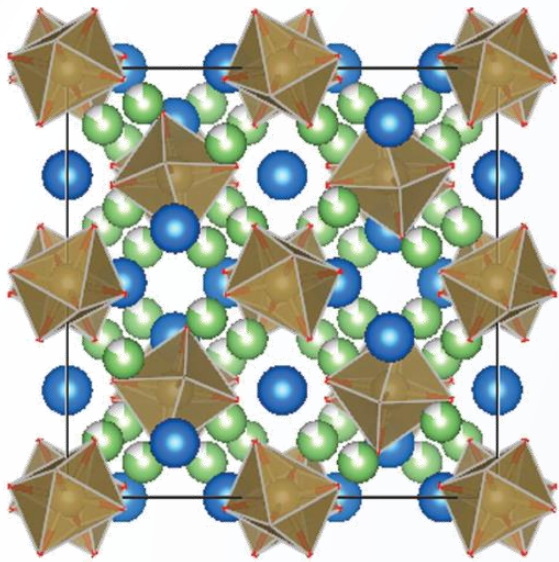
Formula	MPID	Category
$\text{Li}_7\text{P}_3\text{S}_{11}$	mp-641703	thio-LISICON
$\text{Li}_{10}\text{GeP}_2\text{S}_{12}$	mp-696128	thio-LISICON
$\text{Li}_6\text{PS}_5\text{Cl}$	mp-985592	Argyrodite
$\text{Li}_6\text{PS}_5\text{Br}$	mp-985591	Argyrodite
$\text{Li}_6\text{PS}_5\text{I}$	mp-985582	Argyrodite
Li_3AlF_6	mp-15254	Group IIIA Halide
Li_3GaF_6	mp-15558	Group IIIA Halide
$\text{LiZr}_2\text{P}_3\text{O}_{12}$	mp-10499	Li-NASICON
$\text{LiTi}_2\text{P}_3\text{O}_{12}$	mp-18640	Li-NASICON
$\text{Li}_4\text{P}_2\text{O}_7$	mp-554577	Oxide
Li_2SO_4	mp-4556	Oxide
Li_4GeO_4	mp-4558	Oxide

Methods – Speed and accuracy

- AIMD are accurate
- Bond valence methods are fast
 - Migration barriers from pathway analyses (BVPA) are correlated to ionic conductivity
 - Ionic conductivity requires Kinetic Monte Carlo (KMC)
 - softBV
- Machine-learned force fields
 - Require first-principles data for fitting
 - Machine-learned molecular dynamics (MLMD)
 - Ultra-Fast Force Fields (UF³)



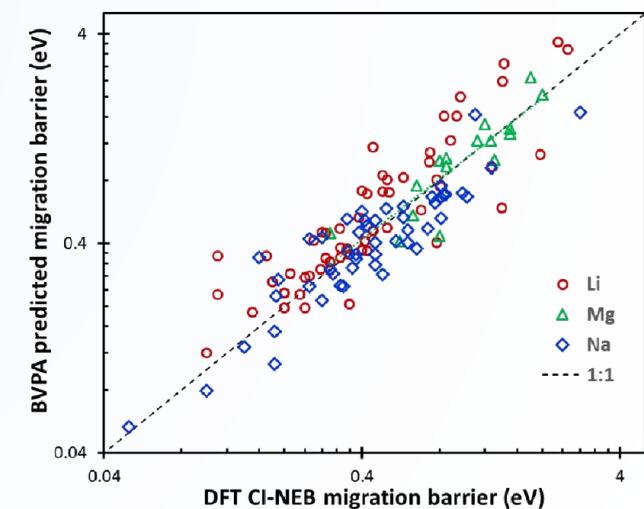
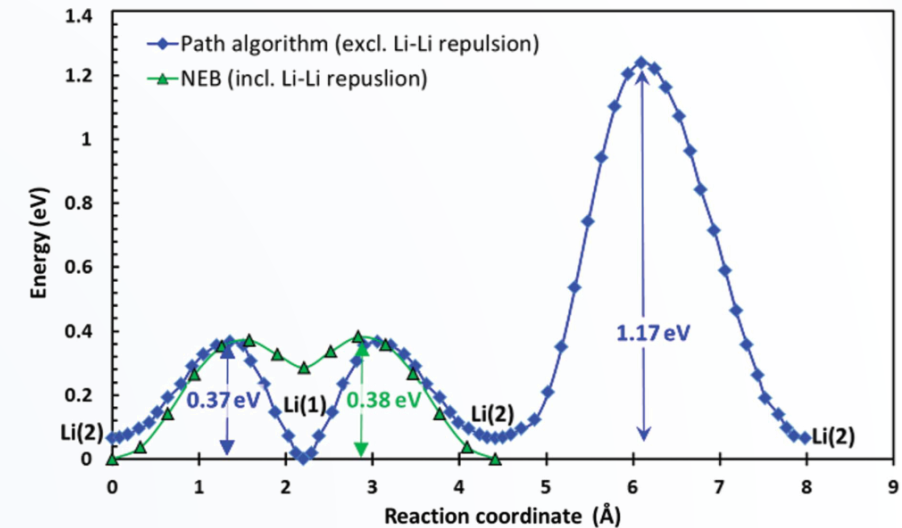
Bond valence methods with softBV



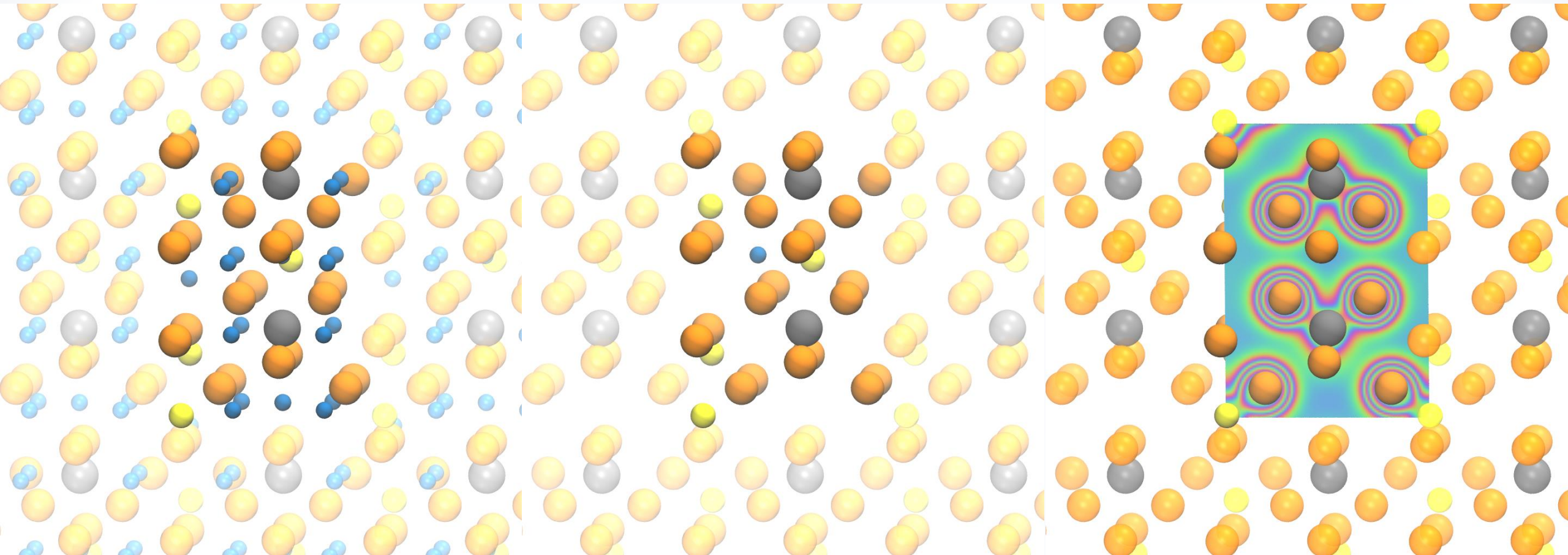
$$E_{\text{bond}} = D_0 \left[\exp \left(\frac{R_{\text{min}} - R}{b} \right) - 1 \right]^2 - D_0$$

$$E_{\text{Coul}} = \frac{q_1 q_2}{R} \operatorname{erfc} \left[\frac{R}{f(r_1 + r_2)} \right]$$

Excellent agreement with migration barriers from climbing-image nudged elastic band calculations (DFT CI-NEB)

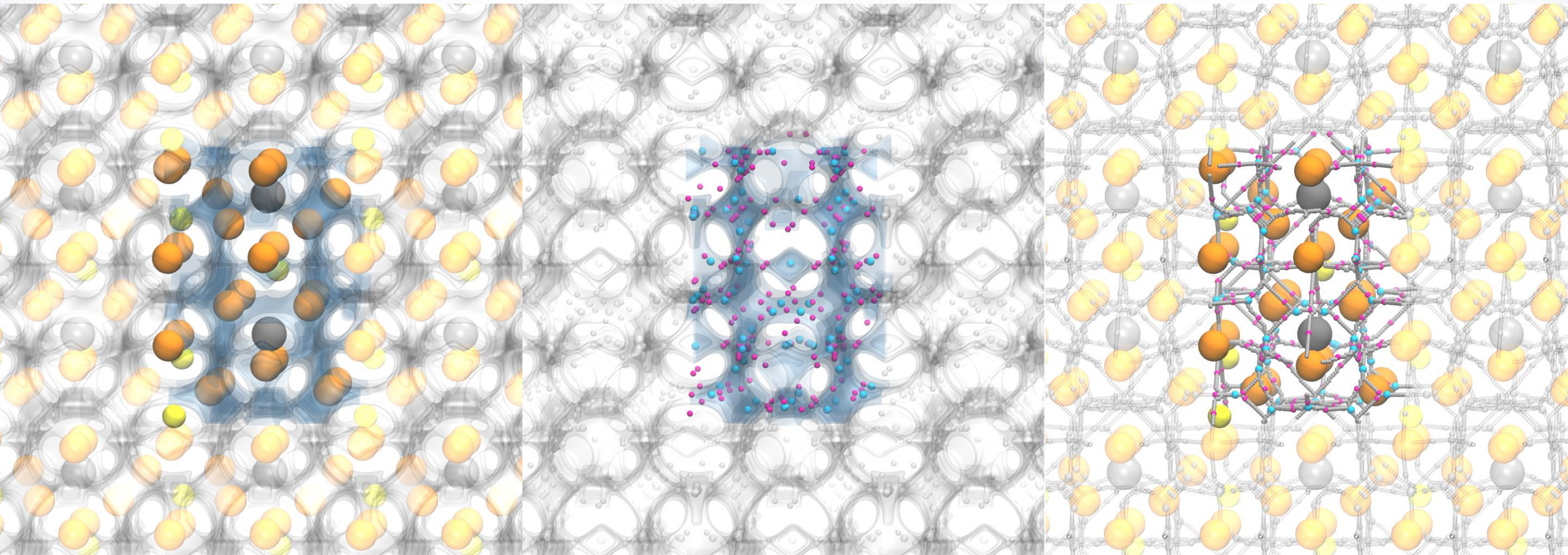


Bond valence methods with softBV



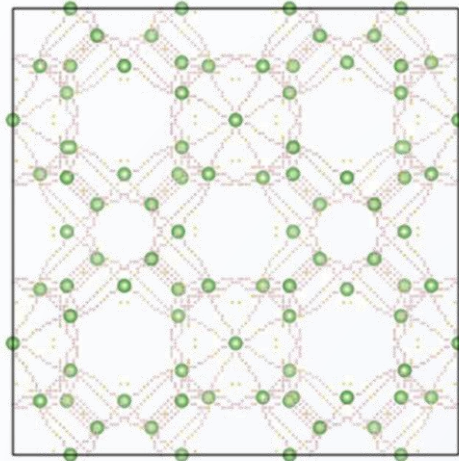
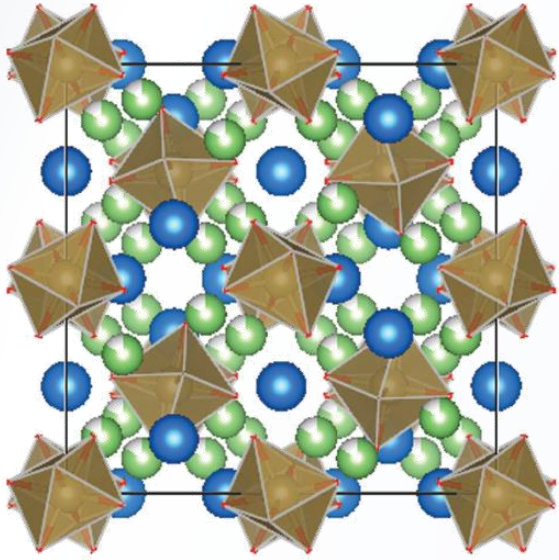
- Energy landscape of single Li atom sampled uniformly with bond valence force field
- Non-Li atoms are immobile, Li-Li interaction ignored

Bond valence methods with softBV



- Low-energy regions reveal percolation pathways for Li
- Pathways connect saddle points and local minima in energy landscape

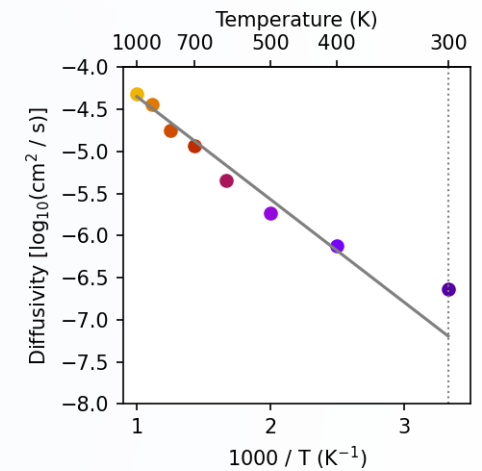
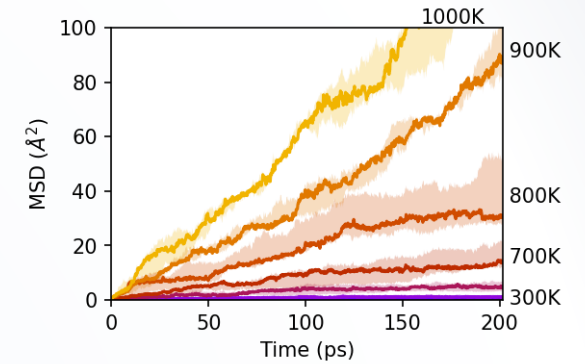
Bond valence methods with softBV



$$p = \omega \exp\left(-\frac{E_m + \Delta E}{k_B T}\right)$$

$$D = \frac{1}{6} \log_{t \rightarrow \infty} \left\langle \left[\overrightarrow{r(t)} - \overrightarrow{r(0)} \right]^2 \right\rangle$$

$$\sigma = \frac{\eta q^2}{k_B T} D$$



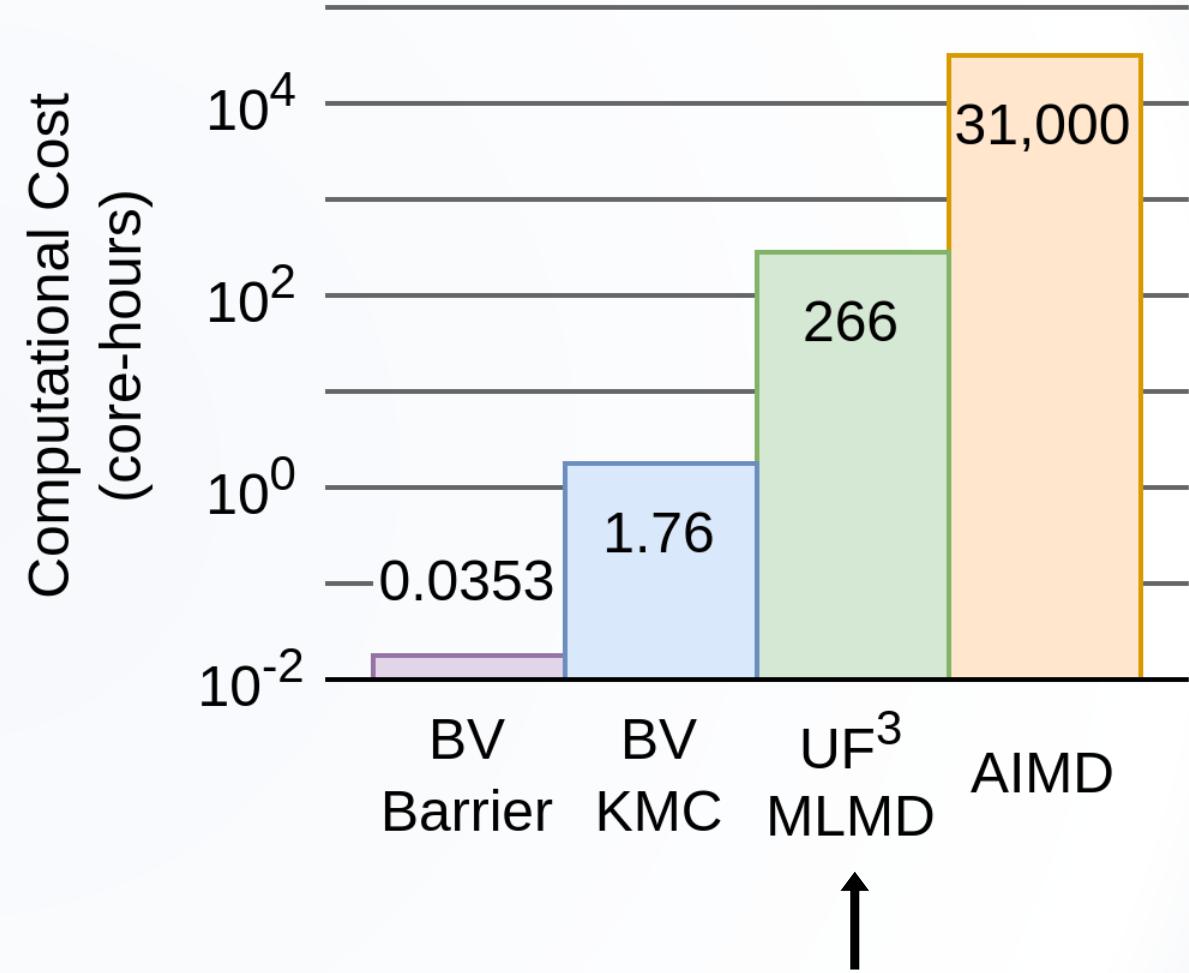
Kinetic Monte Carlo (KMC)

- 1) Construct graph where vertices are interstitial or equilibrium Li sites
- 2) Generate supergraph such that lattice parameters are $\geq 25 \text{ \AA}$
- 3) Distribute Li (set occupancy to 1.0) to sites with lowest energies
- 4) Equilibrate and run the KMC, adjusting migration barriers based on repulsion from instantaneous arrangement of Li on nearby sites
- 5) Fit diffusion constant from MSD
- 6) Compute ionic conductivity from diffusion constant

- p : transition rate for hops to vacant, adjacent sites
- ω : attempt frequency
- E_m : migration barrier
- ΔE : local Li-Li repulsion

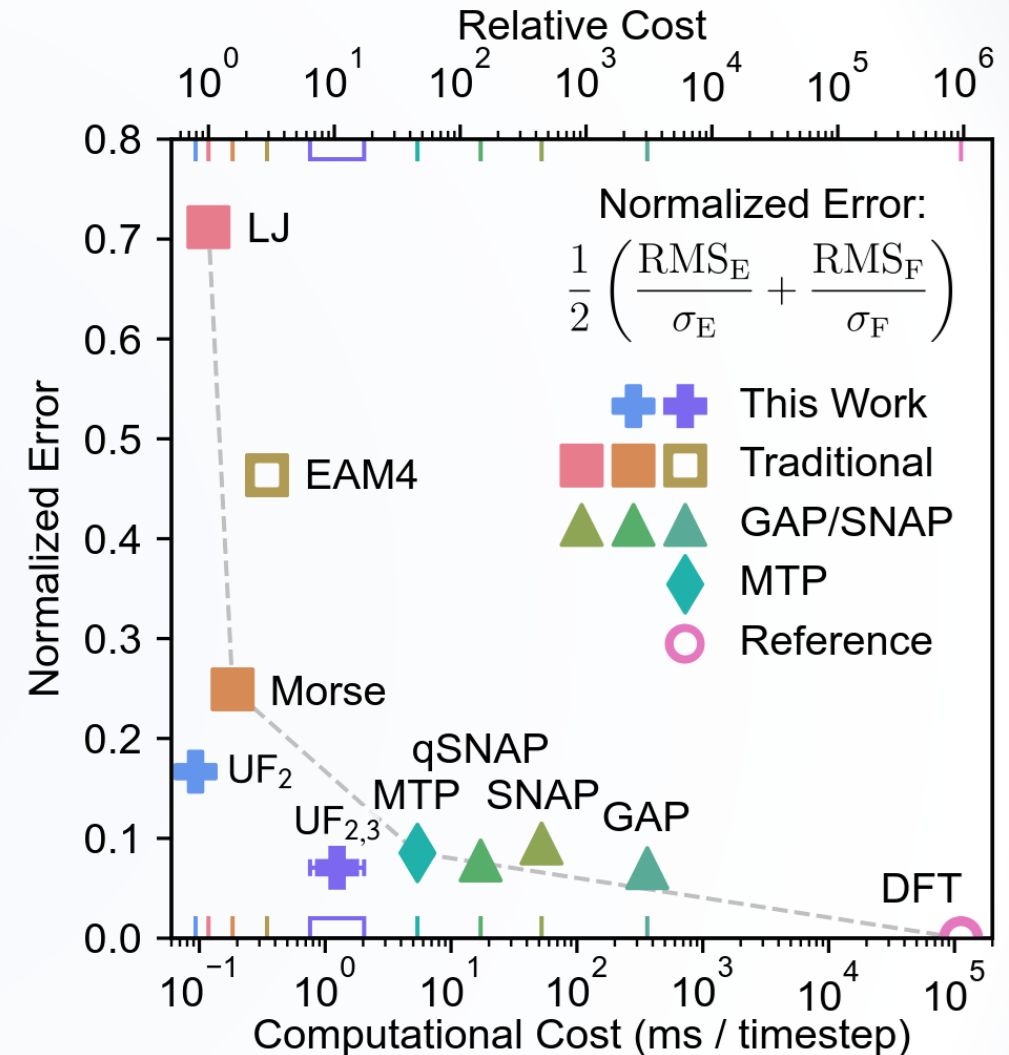
Methods – Speed and accuracy

- AIMD are accurate
- Bond valence methods are fast
 - Migration barriers from pathway analyses (BVPA) are correlated to ionic conductivity
 - Ionic conductivity requires Kinetic Monte Carlo (KMC)
 - softBV
- Machine-learned force fields
 - Require first-principles data for fitting
 - Machine-learned molecular dynamics (MLMD)
 - Ultra-Fast Force Fields (UF³)



Ultra-Fast Force Fields (UF³)

- As fast as empirical potentials
- Accurate enough for applications
- Robust
- Interpretable
- Easy to parametrize



Tungsten; Szlachta et al., PRB, 2014

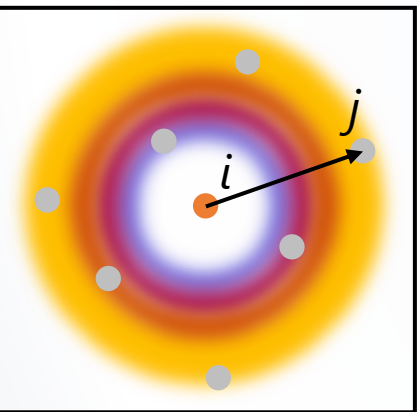
Ultra-Fast Force Fields (UF³)

- Effective two-and-three body potential
- Cubic B-spline basis set
 - Local support
 - Smooth 1st derivative
 - Continuous 2nd derivative

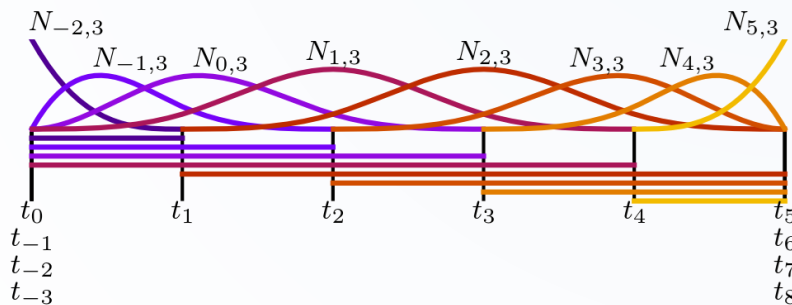
$$E = \sum_{i < j} V_2(r_{ij}) + \sum_{i < j < k} V_3(r_{ij}, r_{ik}, r_{jk})$$

$$V_2(r_{ij}) = \sum_{n=-2}^{K-1} c_n N_{n,3}(r_{ij}) \quad r_{ij} = \sqrt{(R_i - R_j)^2}$$

$$V_3(r_{ij}, r_{ik}, r_{jk}) = \sum_{l=-3}^{K_l-1} \sum_{m=-3}^{K_m-1} \sum_{n=-3}^{K_n-1} c_{lmn} \left(N_{l,3}(r_{ij}) N_{m,3}(r_{ik}) N_{n,3}(r_{jk}) \right)$$



$$N_{n,d+1}(r) = \frac{r - t_n}{t_{n+d} - t_n} N_{n,d}(r) + \frac{t_{n+d+1} - r}{t_{n+d+1} - t_{n+1}} N_{n+1,d}(r) \quad N_{n,1}(r) = \begin{cases} 1, & r \in [t_n, t_{n+1}] \\ 0, & \text{otherwise} \end{cases}$$

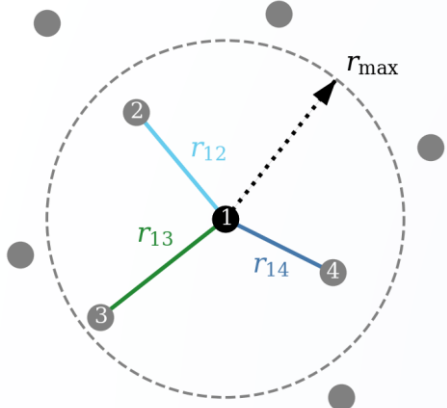


B-splines are defined recursively.
Cubic B-splines have degree $d=3$.

The UF potential is a sum of cubic B-spline basis functions, weighted by spline coefficients c

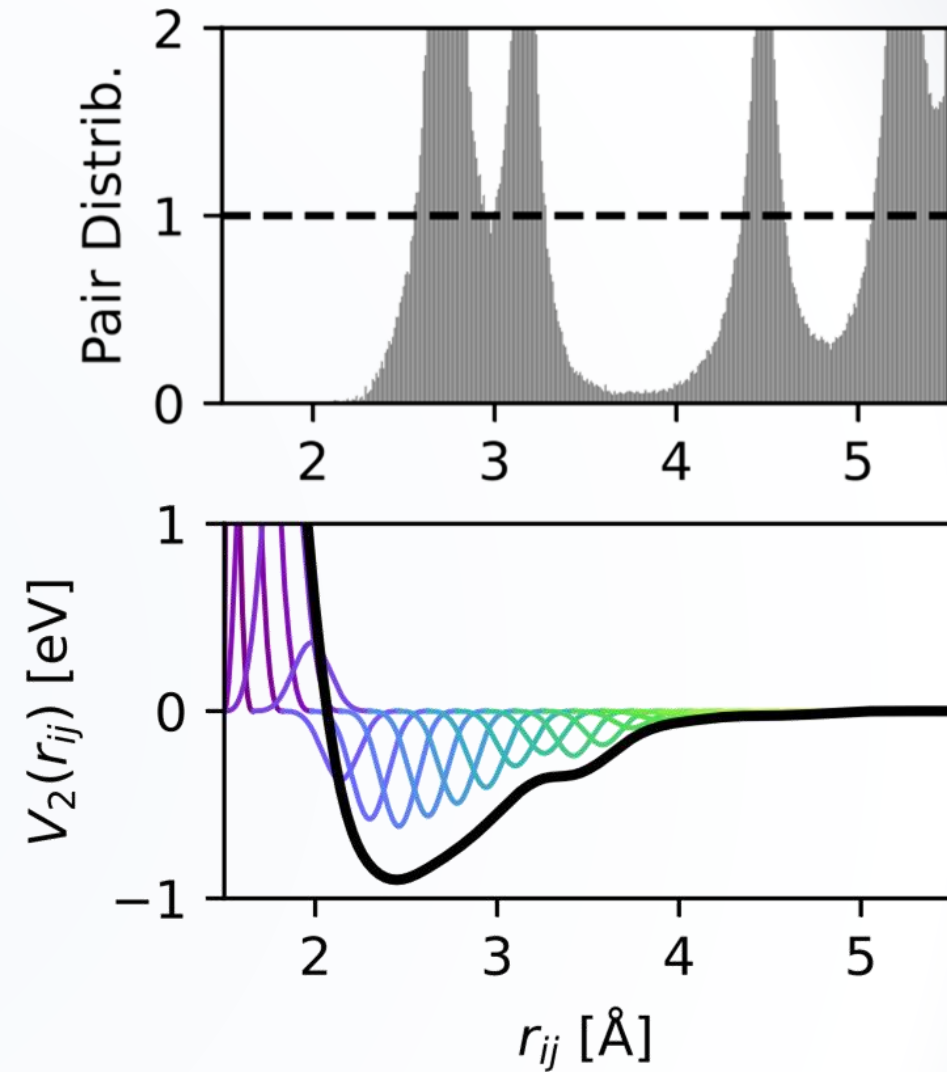
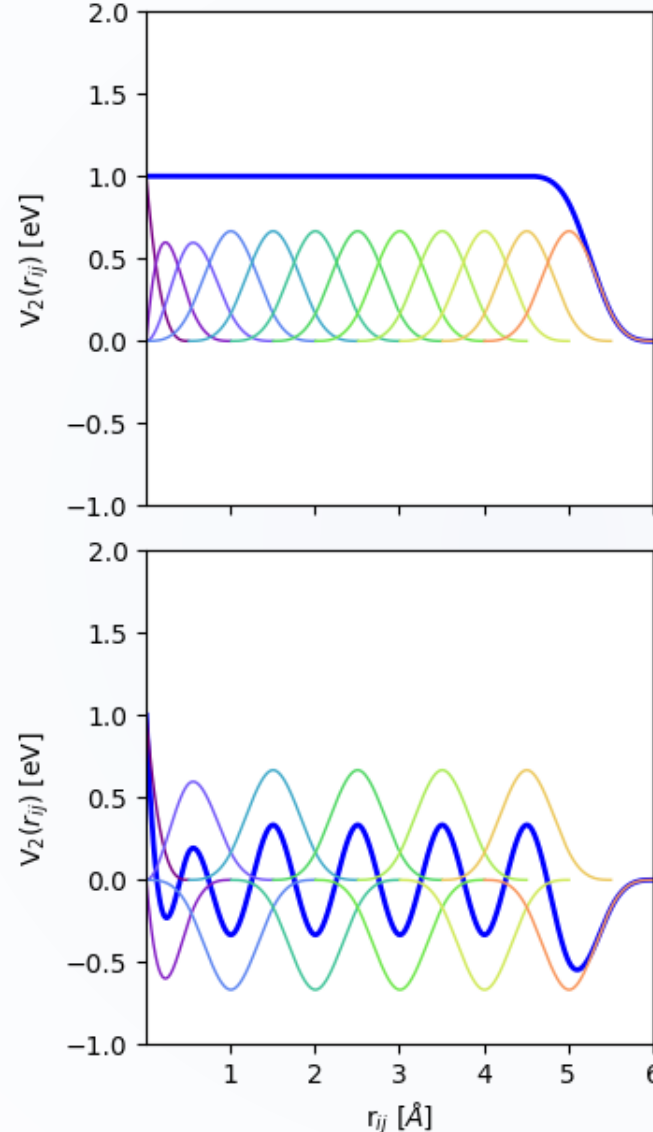
Ultra-Fast Force Fields (UF³)

$$E_{2,1} = V_2(r_{12}) + V_2(r_{13}) + V_2(r_{14})$$



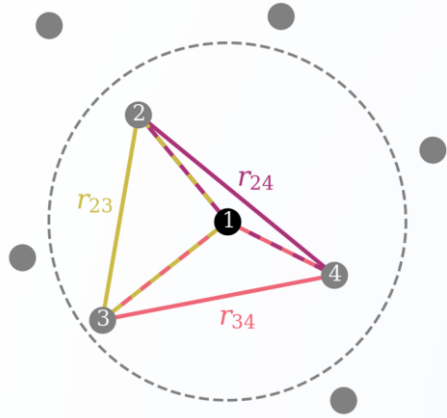
$$V_2(r_{ij}) = \sum_{n=-2}^{K-1} c_n N_{n,3}(r_{ij})$$

- Number of basis functions controls maximum local curvature of the b-spline
- Smooth cutoff(s) by constraining coefficients



Tungsten, 2-body: attractive and repulsive regions; Potential minimum is close to the known nearest-neighbor distance (2.7 Å).

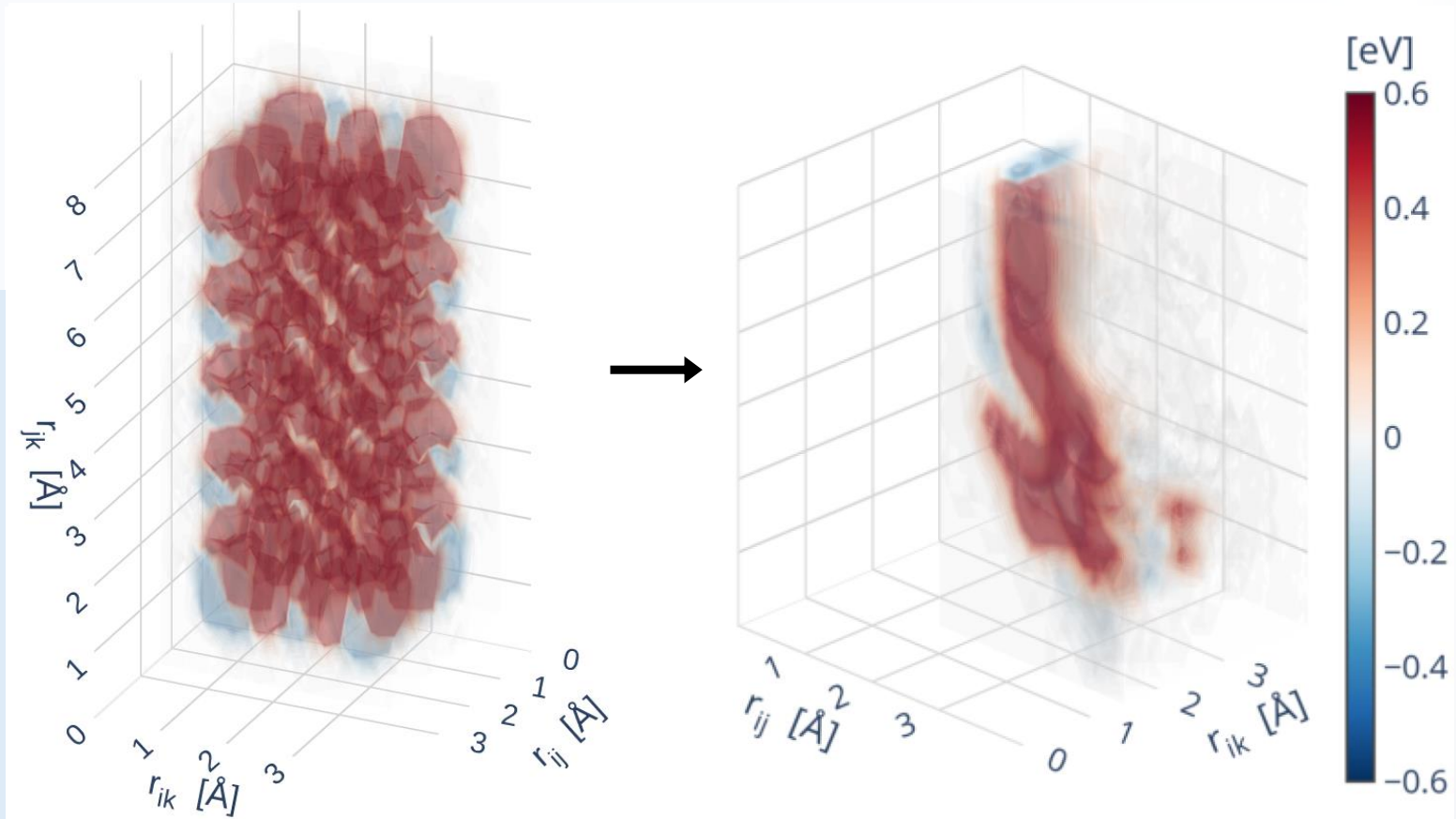
Ultra-Fast Force Fields (UF³)



$$E_{3,1} = V_3(r_{12}, r_{13}, r_{23}) + V_3(r_{12}, r_{14}, r_{24}) + V_3(r_{13}, r_{14}, r_{34})$$

$$V_3(r_{ij}, r_{ik}, r_{jk}) = \sum_{l=-3}^{K_l-1} \sum_{m=-3}^{K_m-1} \sum_{n=-3}^{K_n-1} c_{lmn} \left(N_{l,3}(r_{ij}) N_{m,3}(r_{ik}) N_{n,3}(r_{jk}) \right)$$

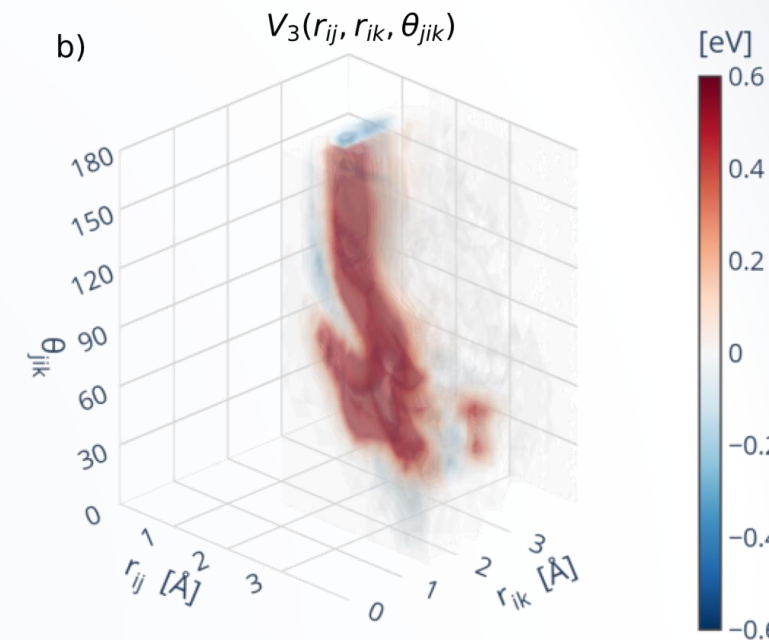
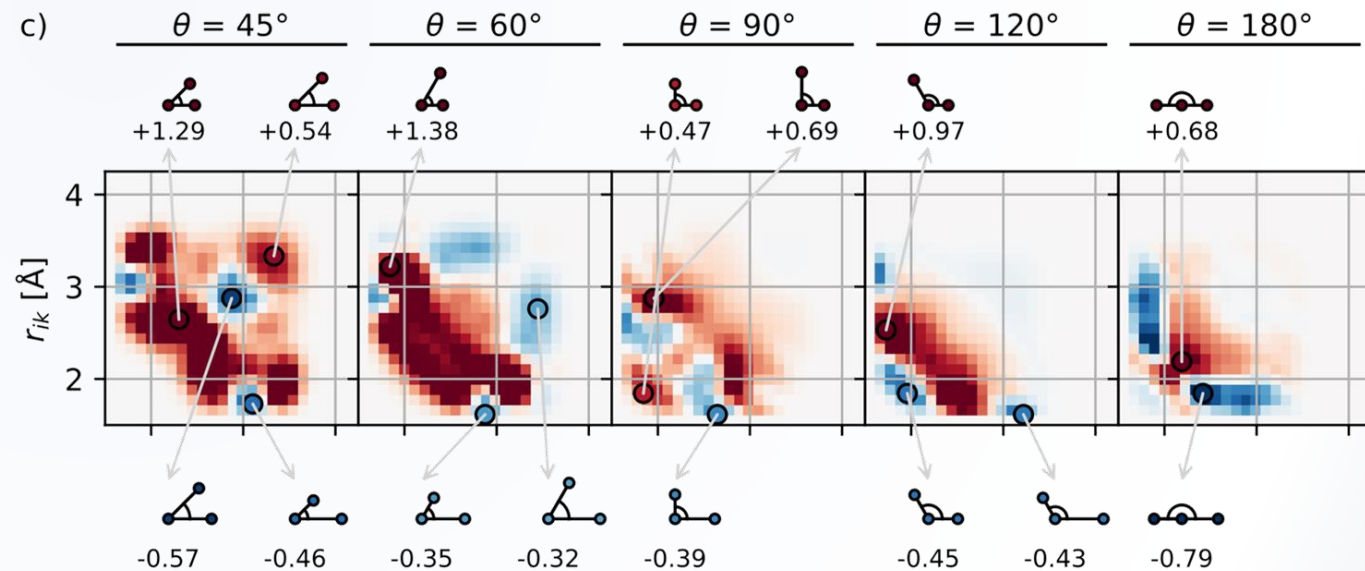
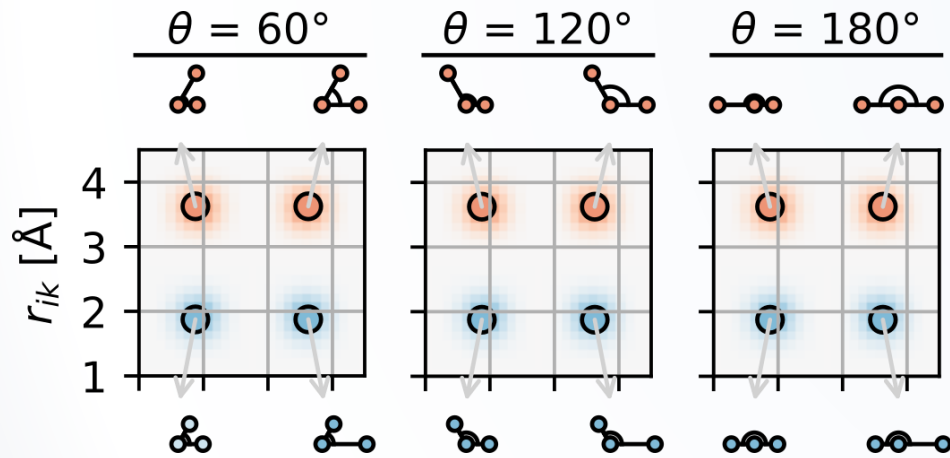
- 3D tensor-product spline
- Fit simultaneously with two-body term during least-squares optimization
- Visualization
 - Blue: favorable interactions
 - Red: unfavorable interactions



Ultra-Fast Force Fields (UF³)

$$V_3(r_{ij}, r_{ik}, r_{jk}) = \sum_{l=-3}^{K_l-1} \sum_{m=-3}^{K_m-1} \sum_{n=-3}^{K_n-1} c_{lmn} \left(N_{l,3}(r_{ij}) N_{m,3}(r_{ik}) N_{n,3}(r_{jk}) \right)$$

- Inspect b-spline function for unphysical “holes”
- Three-body term can be visualized with θ_{jik} instead of r_{jk} for improved interpretability (law of cosines)



Ultra-Fast Force Fields (UF³)

- Inputs (X): B-spline structure representation
- Outputs (y)
 - Potential energy from DFT
 - Interatomic forces from DFT
 - Negative gradient of energy
 - [3 x number of atoms] forces per structure
- Hyperparameters
 - Energy-force tradeoff, κ
 - Regularization strength for two-body and three-body coefficients, λ_2 and λ_3

$$x_{s,n,AB} = \sum_{i \in A} \sum_{j \in B}^{N_s} N_{n,3}(r_{ij})$$

$$x_{s,lmn,ABC} = \sum_{i \in A} \sum_{j \in B} \sum_{k \in C}^{N_s} N_{n,3}(r_{ij}) N_{l,3}(r_{ik}) N_{m,3}(r_{jk})$$

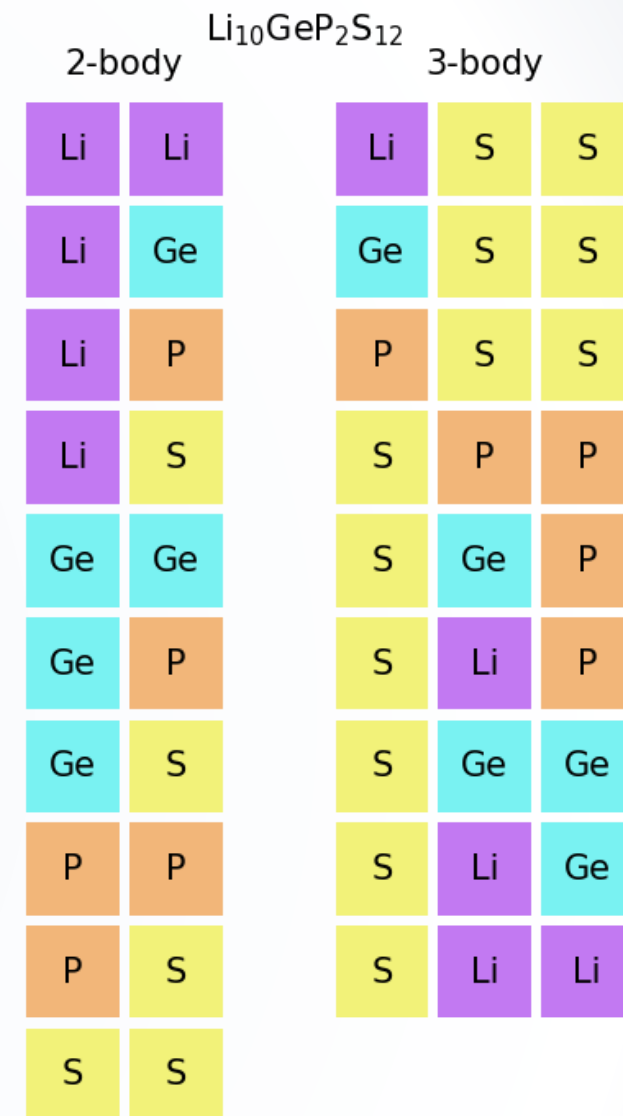
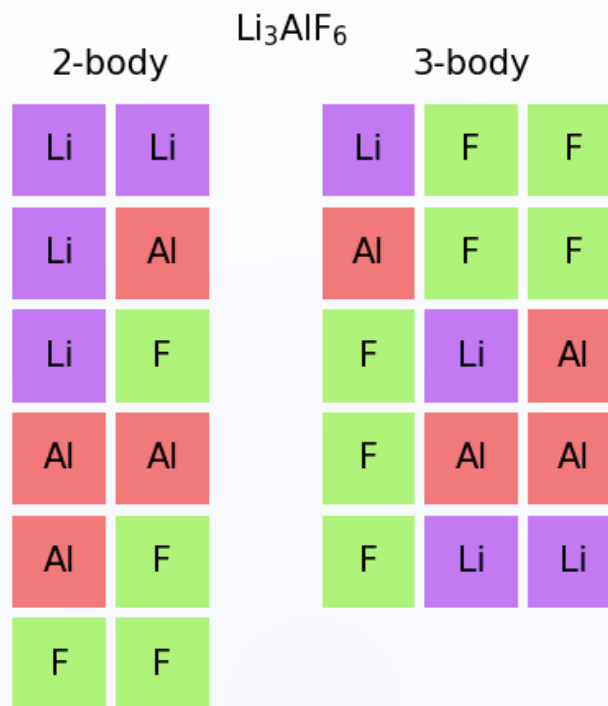
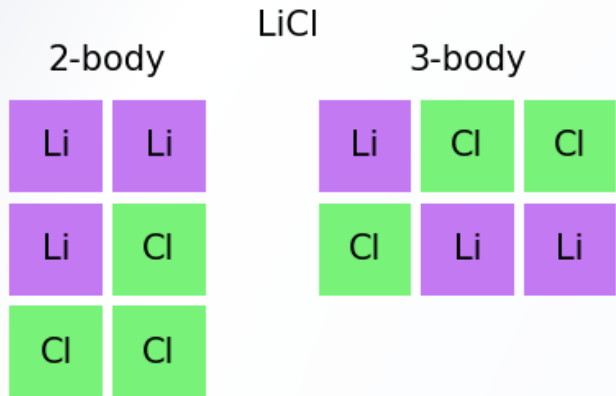
$$Xc = y \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_S \end{bmatrix}$$

$$c = \begin{bmatrix} c_{1,AA} \\ \vdots \\ c_{n,BC} \\ c_{111,AAA} \\ \vdots \\ c_{lmn,CBA} \end{bmatrix}$$

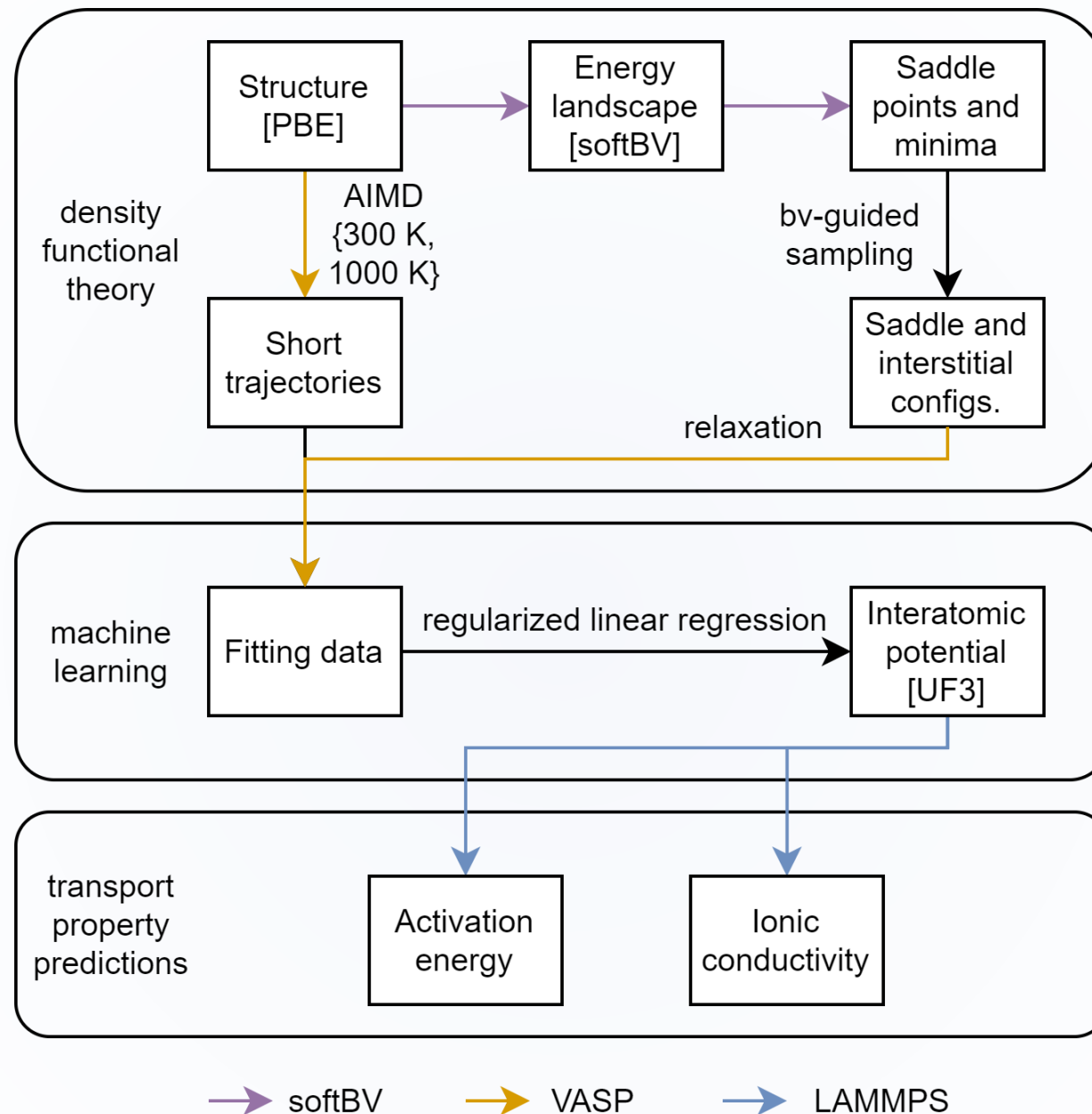
$$X = \begin{bmatrix} x_{1,1,AA} & \cdots & x_{1,n,BC} & x_{1,111,AAA} & \cdots & x_{1,lmn,CBA} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{S,1,AA} & \cdots & x_{S,n,BC} & x_{S,111,AAA} & \cdots & x_{S,lmn,CBA} \end{bmatrix}$$

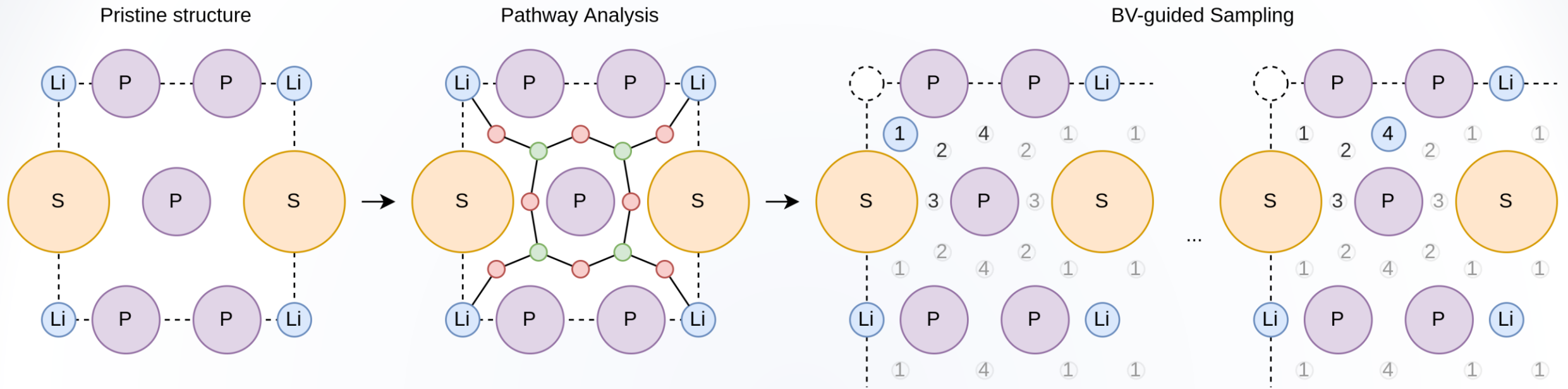
$$\lambda = \begin{bmatrix} \lambda_2 & & & & & 0 \\ & \ddots & & & & \\ & & \lambda_2 & & & \\ & & & \lambda_3 & & \\ & & & & \ddots & \\ 0 & & & & & \lambda_3 \end{bmatrix}$$

$$c = (X^T X + \lambda I)^{-1} X^T y$$

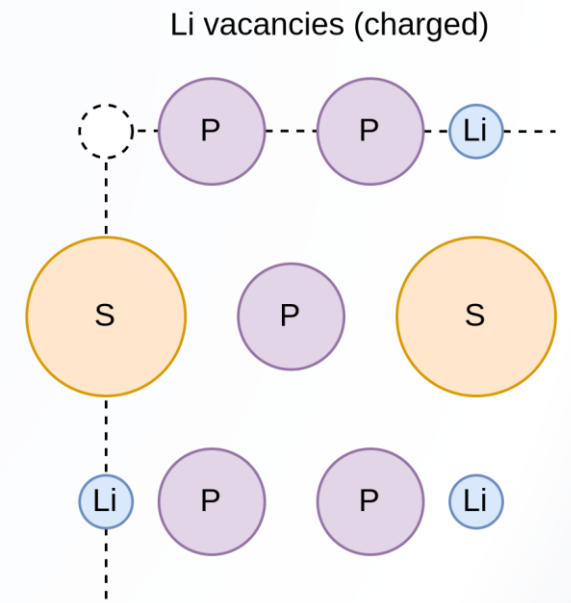


- 2-body:
25 coeff. / interaction
- 3-body:
250 coeff. / interaction
- Reduce fitting complexity and improve data efficiency by ignoring subset of chemical interactions
- Disable cation-anion-cation and anion-cation-anion three-body interactions without losing accuracy



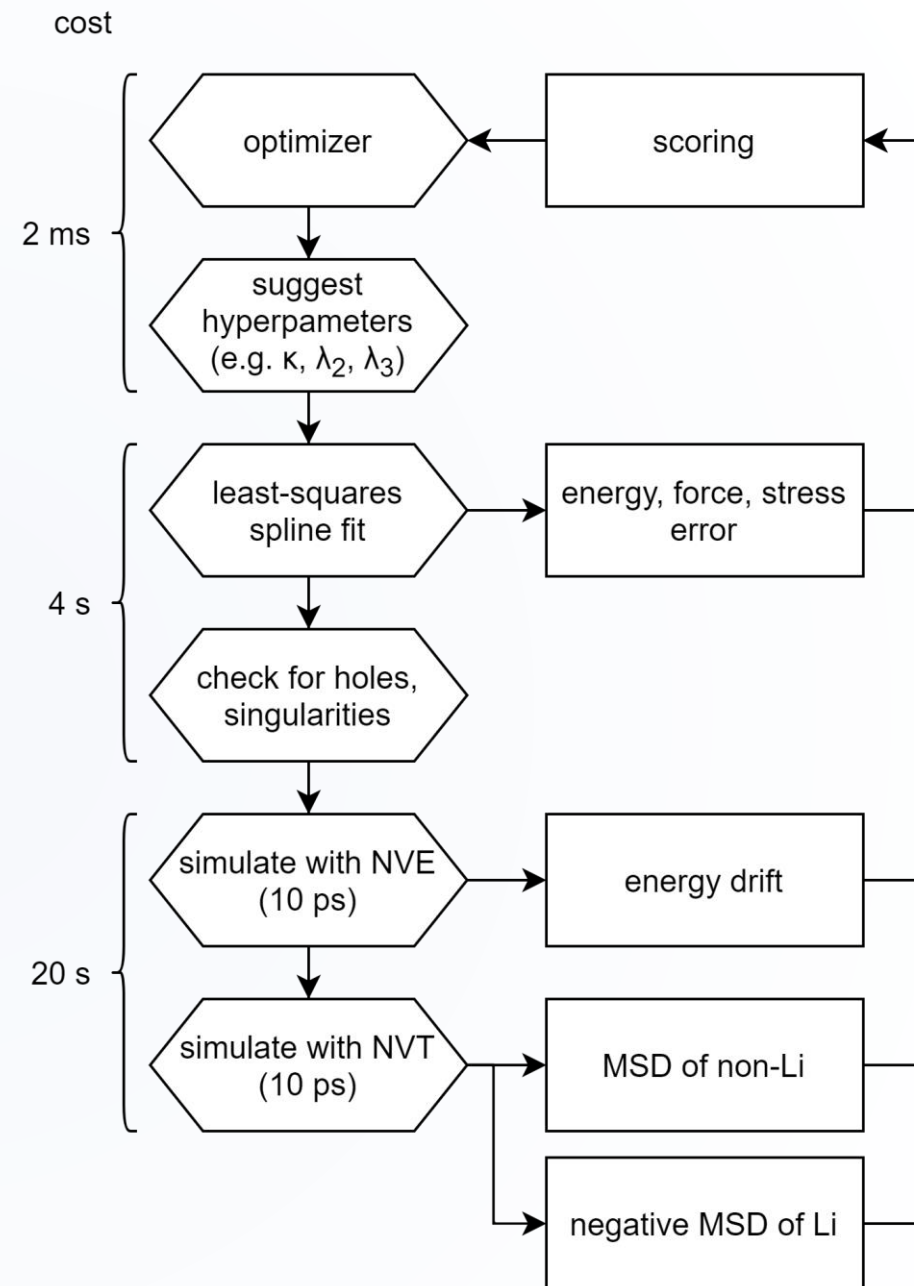


- BV-guided sampling
 - For each saddle point or local minimum in softBV energy landscape (node), identify closest N lithium sites based on BV pathways
 - For each site-node pair, generate a configuration by moving the Lithium from the site to the node (creating a Frenkel defect)
- Lithium vacancy configurations

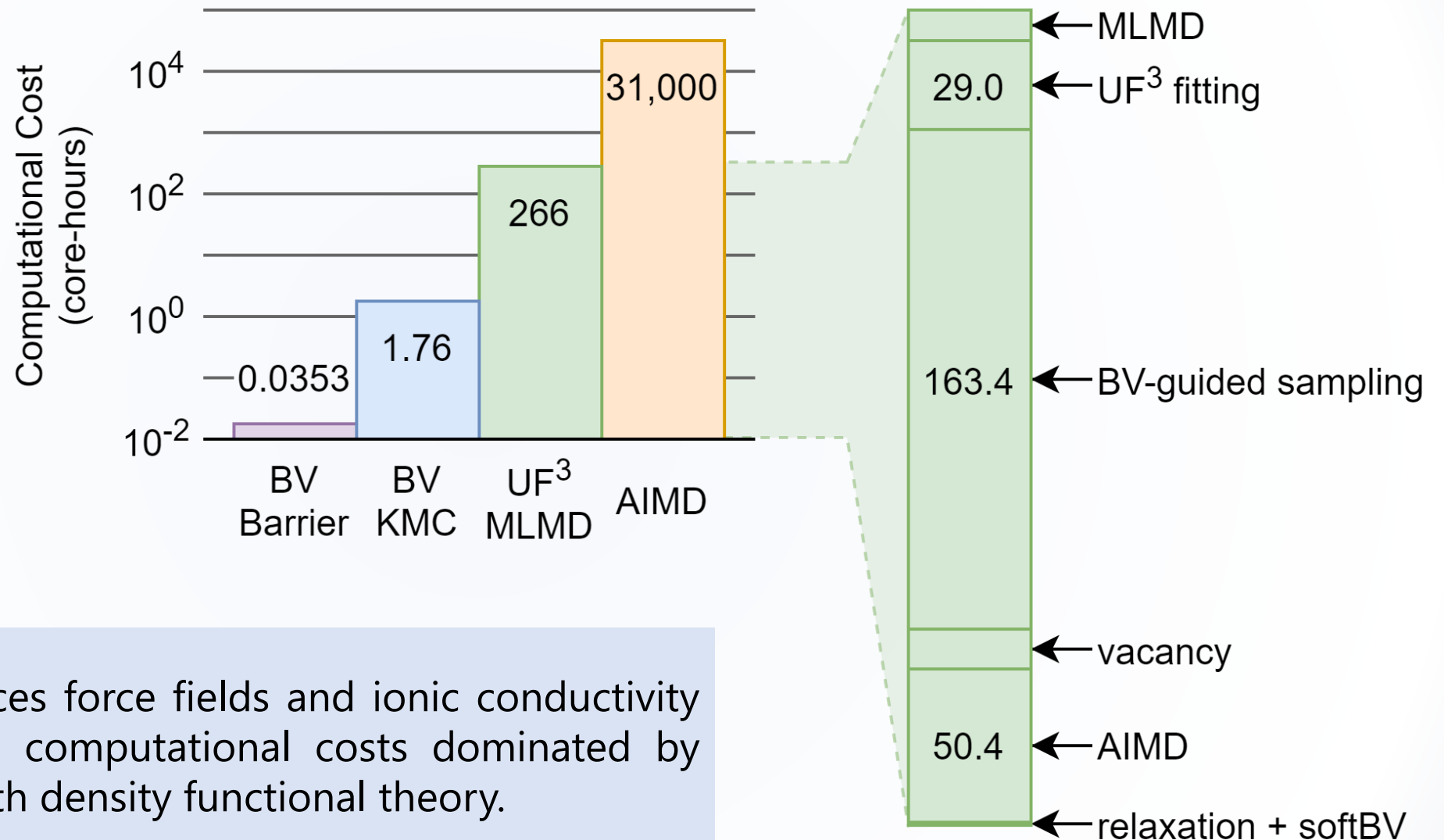


Model fitting

- Optimizer
 - User-assigned weights for single-objective score
 - Multi-objective algorithms
- Least-squares optimization
 - B-spline coefficients
 - Fit DFT energy, forces, and stresses from atomic configurations
- Fast fitting and fast evaluation allows for LAMMPS-in-the-loop optimization
 - Conservation of energy
 - Diffusion behavior

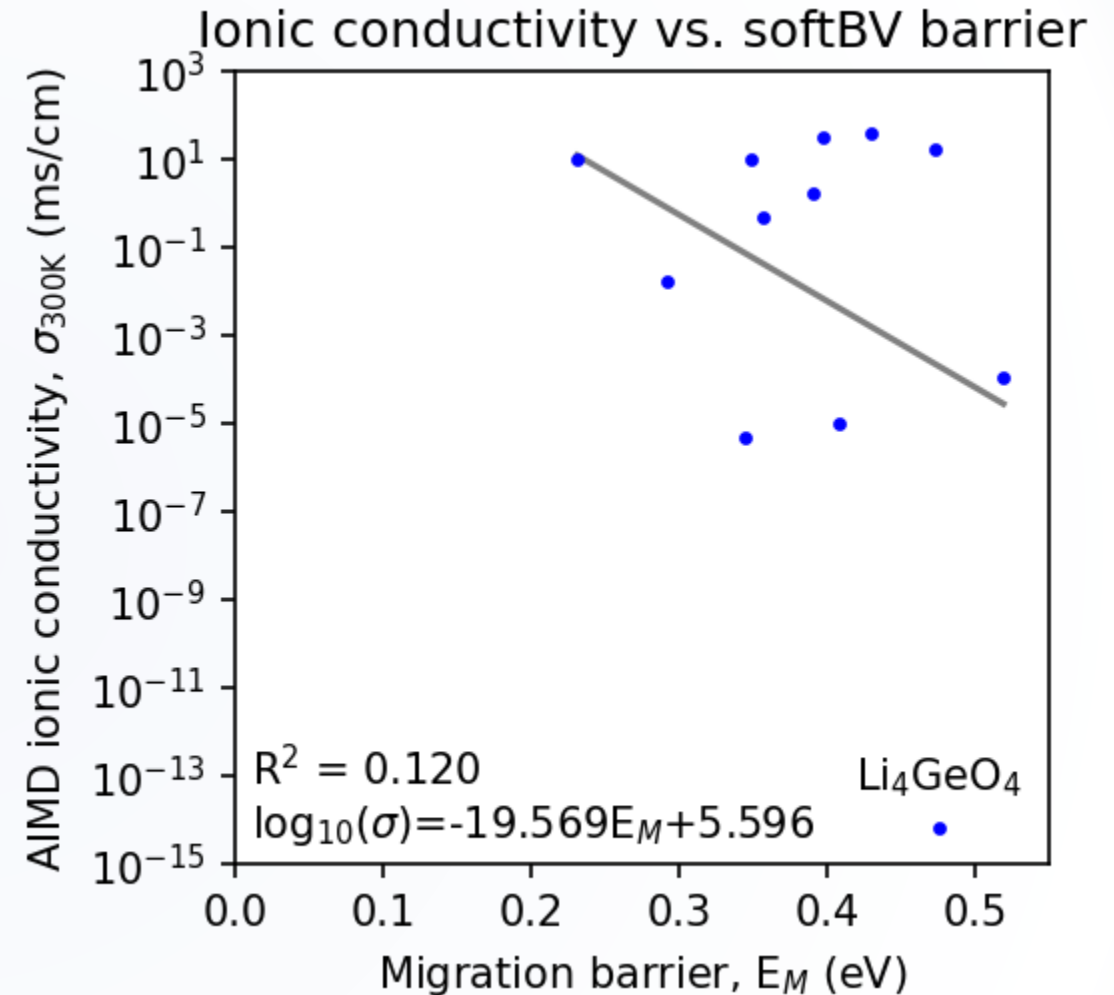
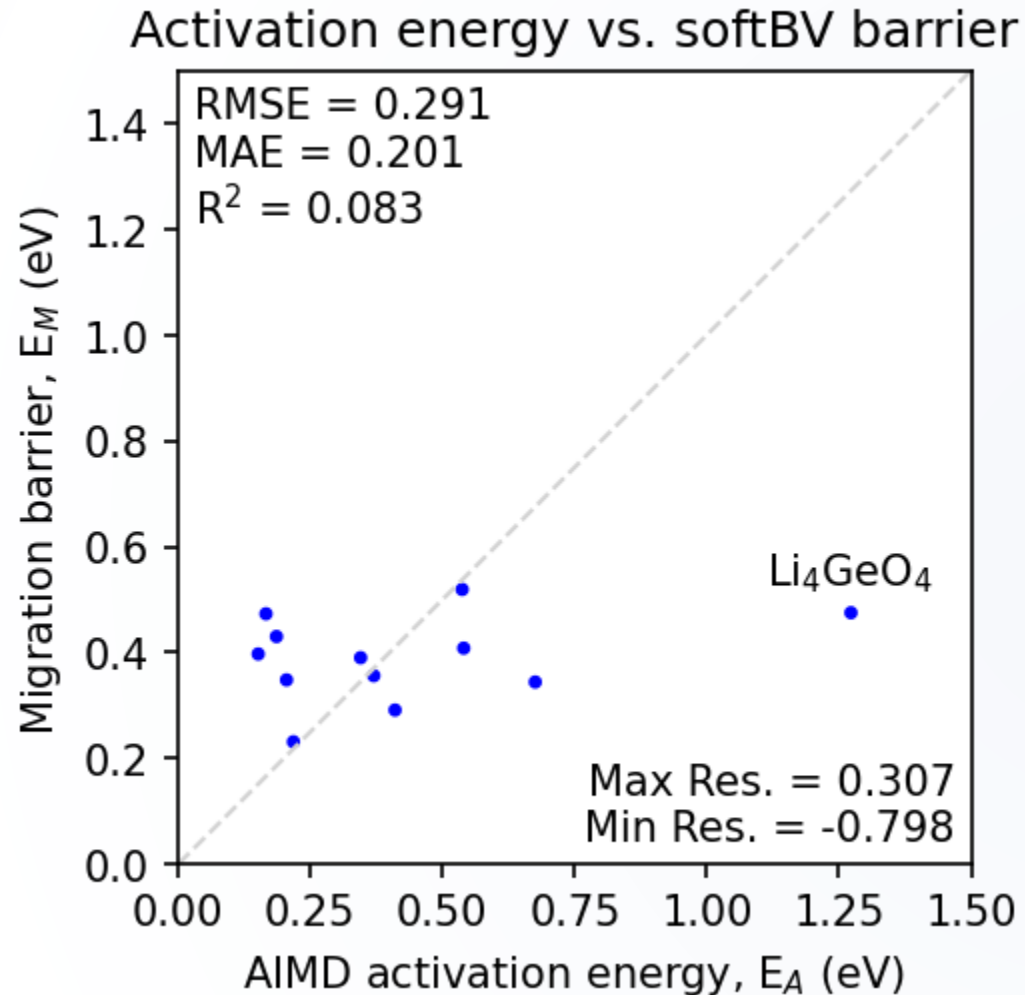


Computational cost



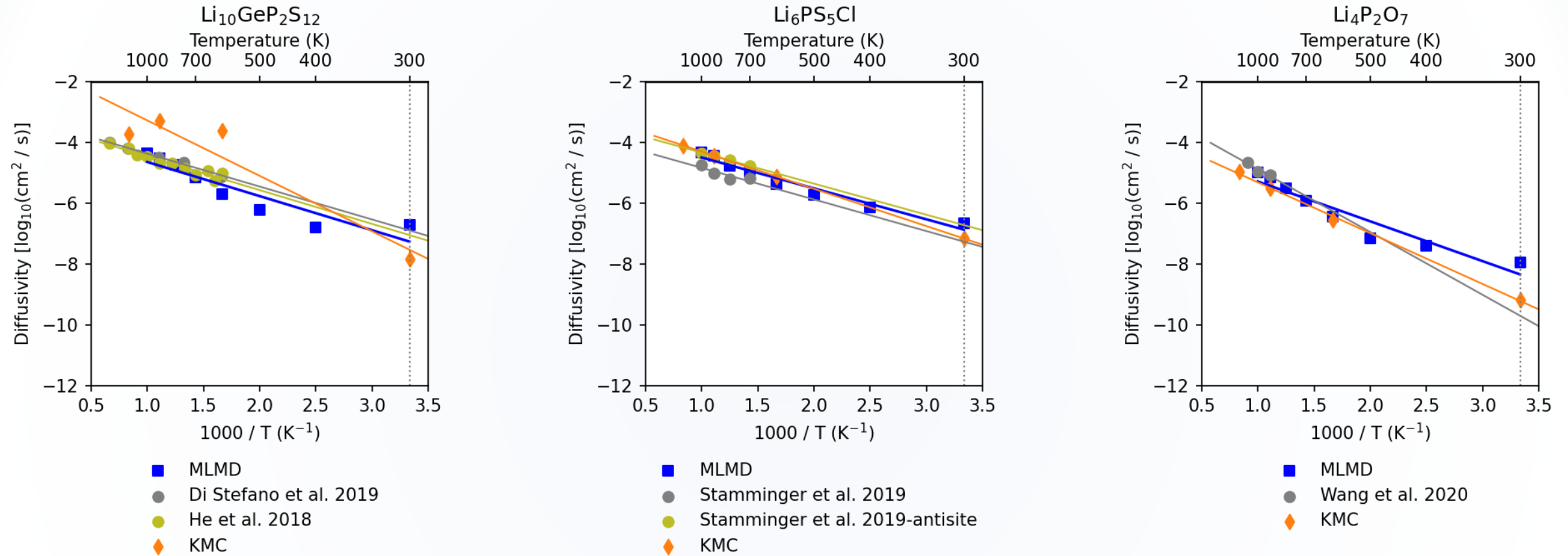
The UF3 framework produces force fields and ionic conductivity predictions with moderate computational costs dominated by generating training data with density functional theory.

softBV migration barrier as a proxy for ionic conductivity



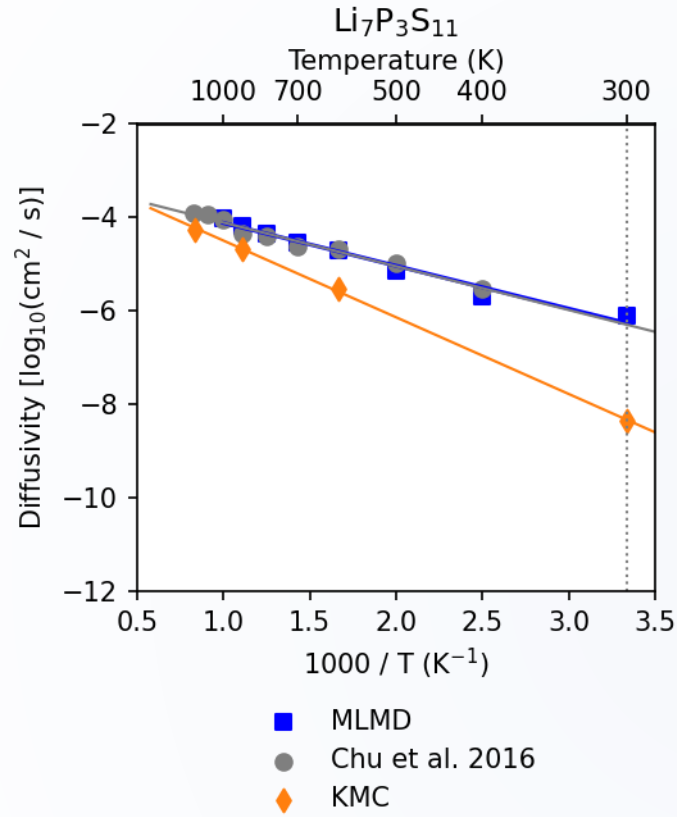
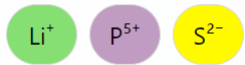
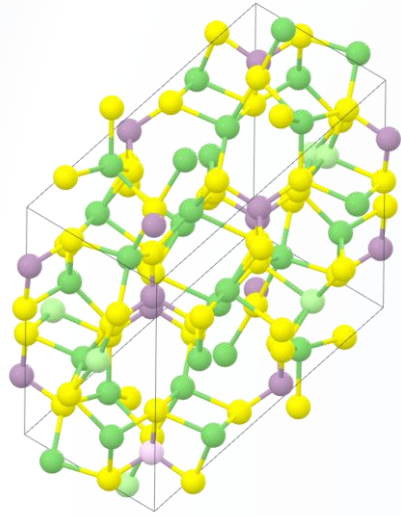
Coarse screening: migration barrier is weakly correlated with both activation energy and ionic conductivity

Predicting room temperature ionic conductivity



For many materials, both softBV (KMC) and UF3 (MD) yield ionic conductivity predictions in agreement with AIMD.

Predicting room temperature ionic conductivity

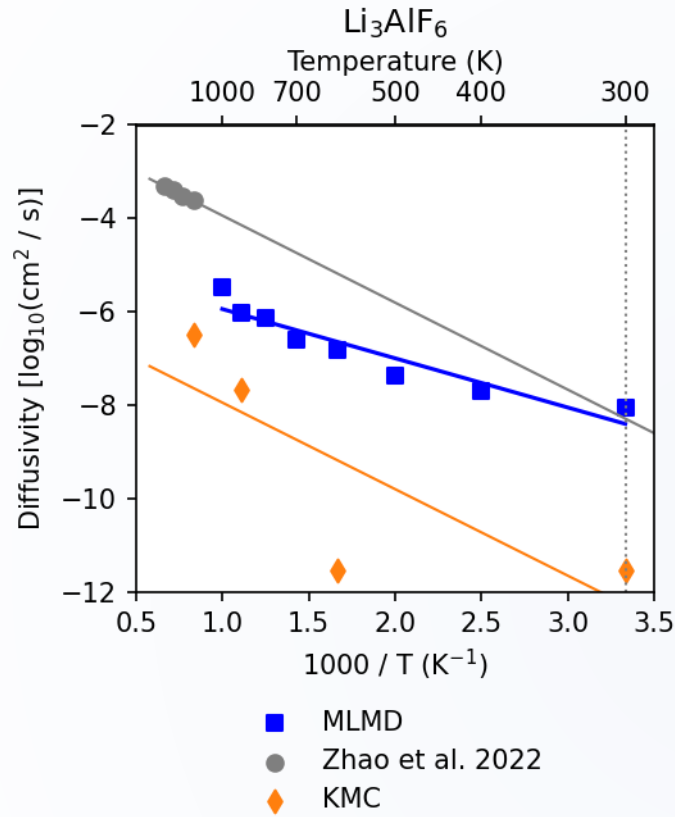
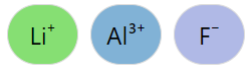
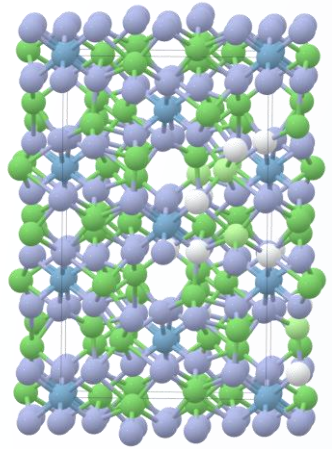


Formula	Density (0.01 \AA^{-3})	Li Density (0.01 \AA^{-3})	Spacegroup Number	BV Minima	BV saddle points
$\text{Li}_7\text{P}_3\text{S}_{11}$	5.0	1.7	2	33	80
$\text{Li}_{10}\text{GeP}_2\text{S}_1$	5.3	2.1	105	15	30
$\text{Li}_6\text{PS}_5\text{Cl}$	5.0	2.3	216	5	7
$\text{Li}_6\text{PS}_5\text{Br}$	5.0	2.3	216	5	8
$\text{Li}_6\text{PS}_5\text{I}$	4.9	2.3	216	5	8
Li_3AlF_6	10.2	3.1	15	16	56
Li_3GaF_6	9.7	2.9	15	16	54
$\text{LiZr}_2\text{P}_3\text{O}_{12}$	7.2	0.4	14	10	25
$\text{LiTi}_2\text{P}_3\text{O}_{12}$	8.1	0.5	167	3	8
$\text{Li}_4\text{P}_2\text{O}_7$	9.1	2.8	14	14	42
Li_2SO_4	8.5	2.4	14	12	39
Li_4GeO_4	10.4	4.6	63	6	11

Outlier: low symmetry, high number of minima

- High density of nodes in KMC graph
- Shorter hops
- Low MSD → low ionic conductivity

Predicting room temperature ionic conductivity

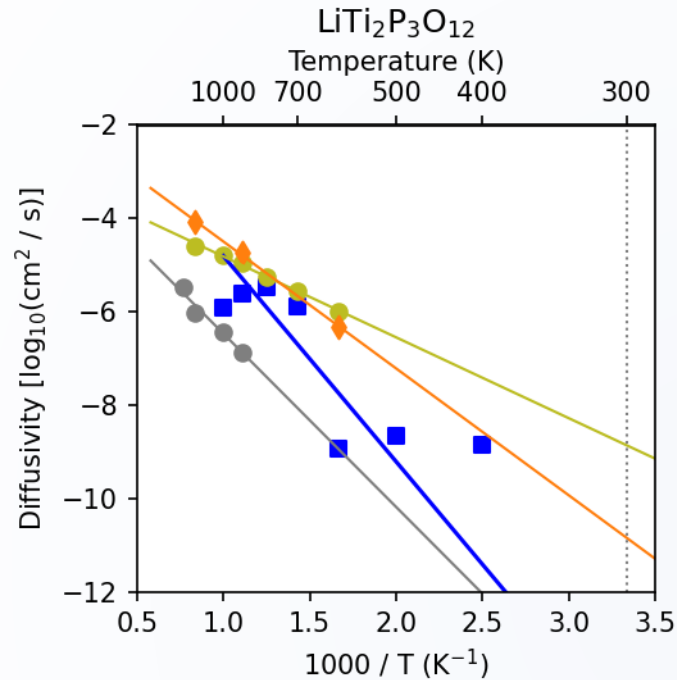
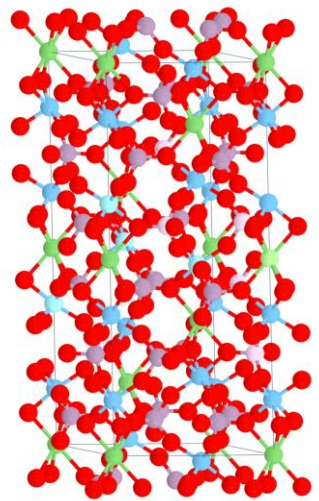


Formula	Density (0.01 Å ⁻³)	Li Density (0.01 Å ⁻³)	Spacegroup Number	BV Minima	BV saddle points
Li ₇ P ₃ S ₁₁	5.0	1.7	2	33	80
Li ₁₀ GeP ₂ S ₁	5.3	2.1	105	15	30
Li ₆ PS ₅ Cl	5.0	2.3	216	5	7
Li ₆ PS ₅ Br	5.0	2.3	216	5	8
Li ₆ PS ₅ I	4.9	2.3	216	5	8
Li ₃ AlF ₆	10.2	3.1	15	16	56
Li ₃ GaF ₆	9.7	2.9	15	16	54
LiZr ₂ P ₃ O ₁₂	7.2	0.4	14	10	25
LiTi ₂ P ₃ O ₁₂	8.1	0.5	167	3	8
Li ₄ P ₂ O ₇	9.1	2.8	14	14	42
Li ₂ SO ₄	8.5	2.4	14	12	39
Li ₄ GeO ₄	10.4	4.6	63	6	11

Outlier: high number density of atoms

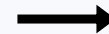
- Dense neighbor lists
- Transferability of softBV parametrization

Predicting room temperature ionic conductivity



- MLMD
- Zhang et al. 2019-vacancy
- Zhang et al. 2019-interstitial
- ◆ KMC

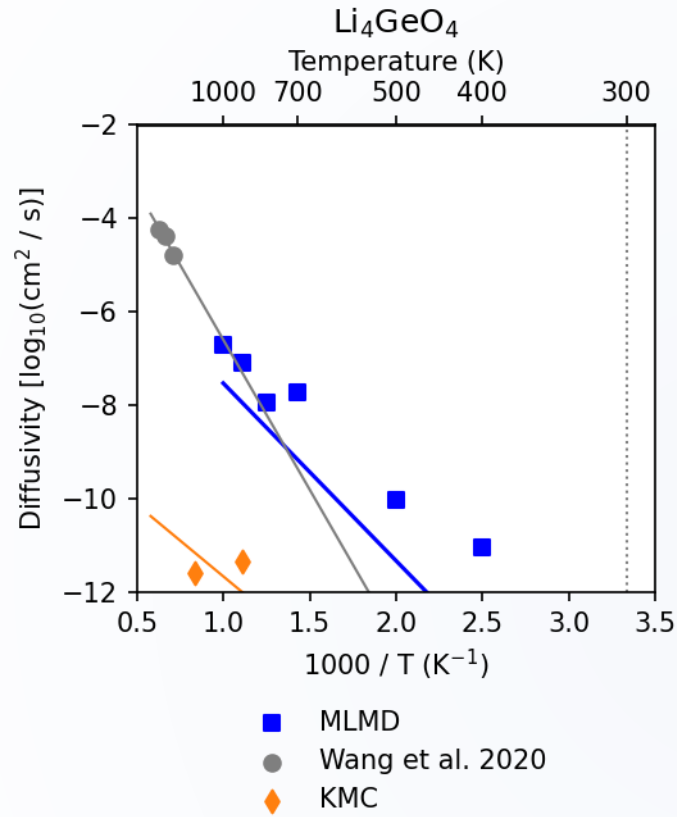
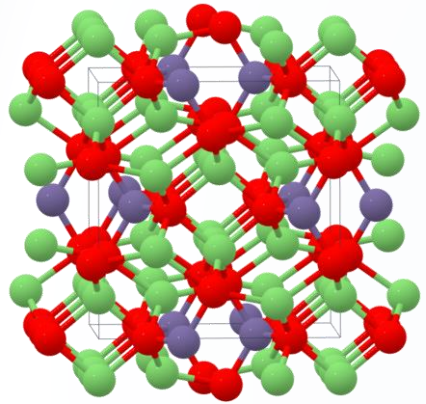
Formula	Density (0.01 \AA^{-3})	Li Density (0.01 \AA^{-3})	Spacegroup Number	BV Minima	BV saddle points
$\text{Li}_7\text{P}_3\text{S}_{11}$	5.0	1.7	2	33	80
$\text{Li}_{10}\text{GeP}_2\text{S}_1$	5.3	2.1	105	15	30
$\text{Li}_6\text{PS}_5\text{Cl}$	5.0	2.3	216	5	7
$\text{Li}_6\text{PS}_5\text{Br}$	5.0	2.3	216	5	8
$\text{Li}_6\text{PS}_5\text{I}$	4.9	2.3	216	5	8
Li_3AlF_6	10.2	3.1	15	16	56
Li_3GaF_6	9.7	2.9	15	16	54
$\text{LiZr}_2\text{P}_3\text{O}_{12}$	7.2	0.4	14	10	25
$\text{LiTi}_2\text{P}_3\text{O}_{12}$	8.1	0.5	167	3	8
$\text{Li}_4\text{P}_2\text{O}_7$	9.1	2.8	14	14	42
Li_2SO_4	8.5	2.4	14	12	39
Li_4GeO_4	10.4	4.6	63	6	11



Outlier: low Li density, low number of minima and saddle points

- Few nodes for bv-guided sampling means less data for ML
- Transferability of softBV parametrization

Predicting room temperature ionic conductivity

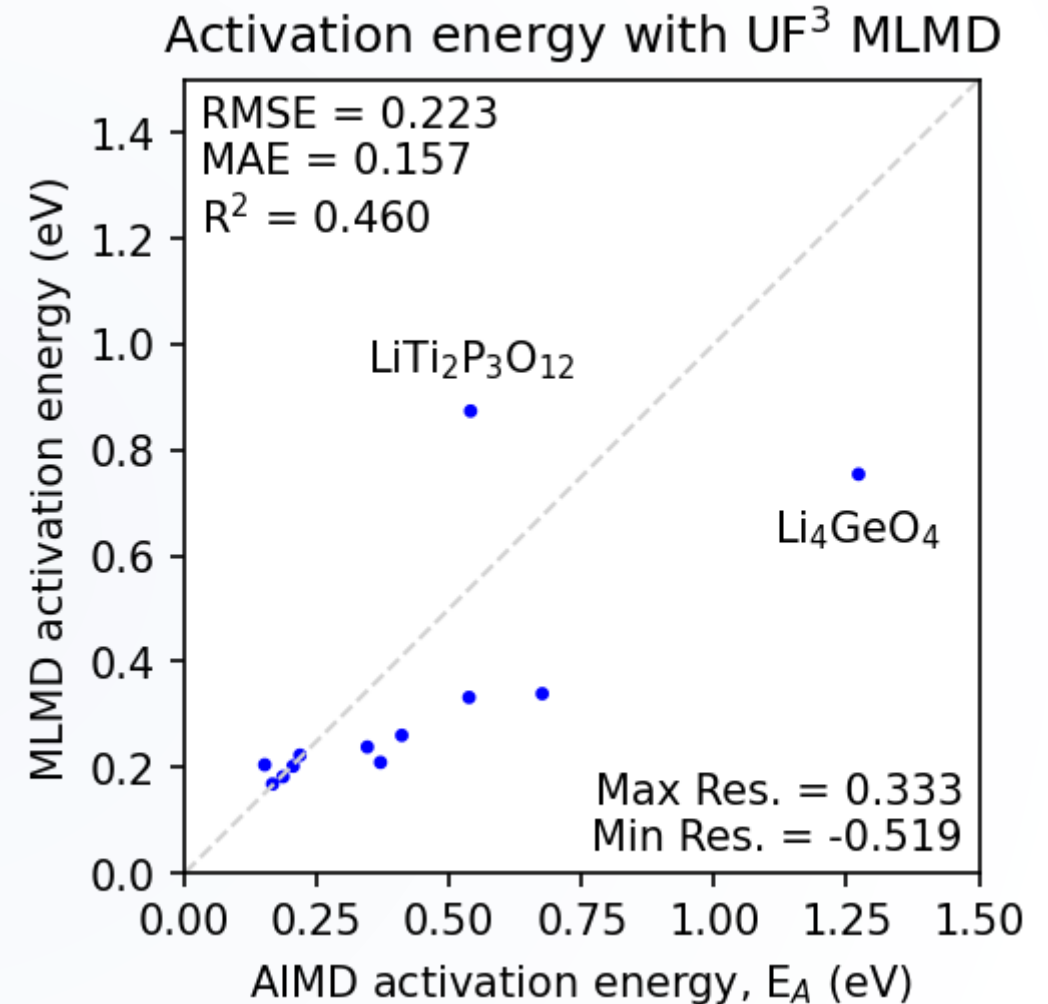
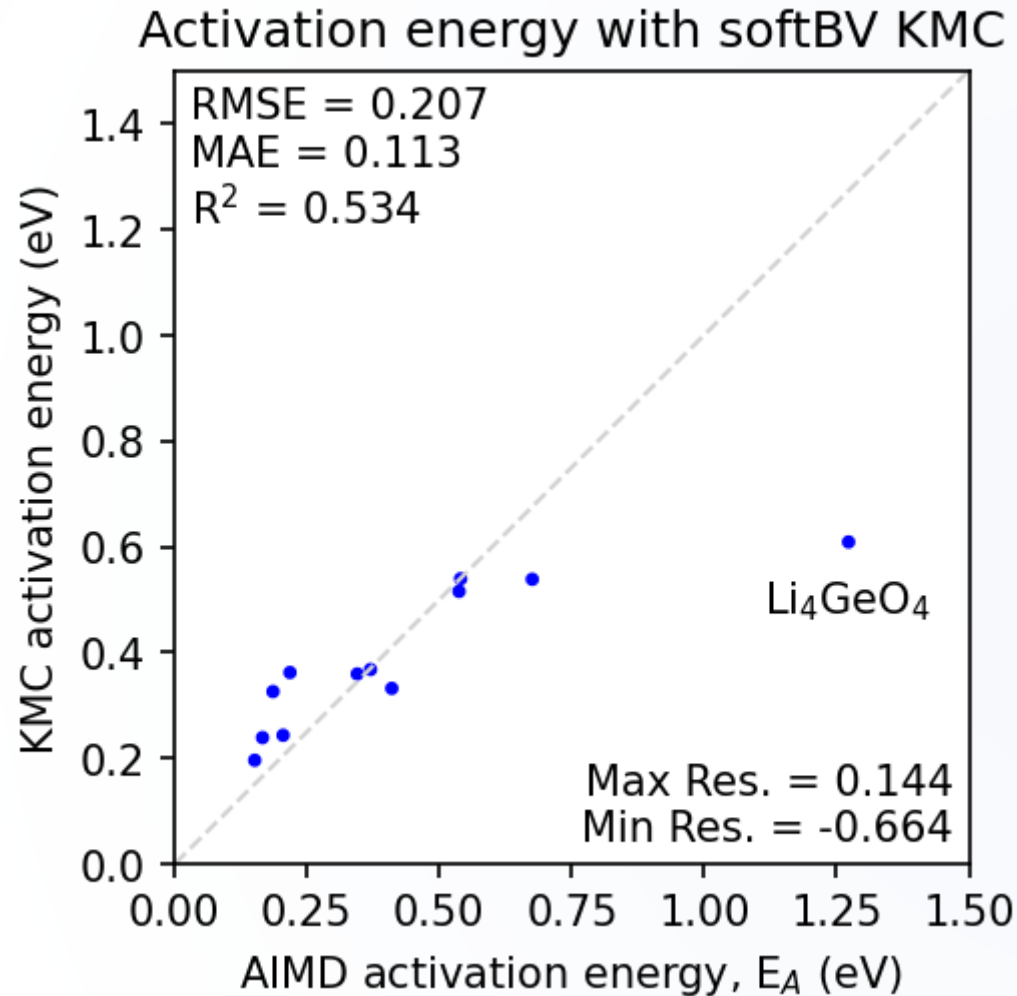


Formula	Density (0.01 \AA^{-3})	Li Density (0.01 \AA^{-3})	Spacegroup Number	BV Minima	BV saddle points
$\text{Li}_7\text{P}_3\text{S}_{11}$	5.0	1.7	2	33	80
$\text{Li}_{10}\text{GeP}_2\text{S}_1$	5.3	2.1	105	15	30
$\text{Li}_6\text{PS}_5\text{Cl}$	5.0	2.3	216	5	7
$\text{Li}_6\text{PS}_5\text{Br}$	5.0	2.3	216	5	8
$\text{Li}_6\text{PS}_5\text{I}$	4.9	2.3	216	5	8
Li_3AlF_6	10.2	3.1	15	16	56
Li_3GaF_6	9.7	2.9	15	16	54
$\text{LiZr}_2\text{P}_3\text{O}_{12}$	7.2	0.4	14	10	25
$\text{LiTi}_2\text{P}_3\text{O}_{12}$	8.1	0.5	167	3	8
$\text{Li}_4\text{P}_2\text{O}_7$	9.1	2.8	14	14	42
Li_2SO_4	8.5	2.4	14	12	39
Li_4GeO_4	10.4	4.6	63	6	11

Outlier: high number density, low ionic conductivity, high activation energy

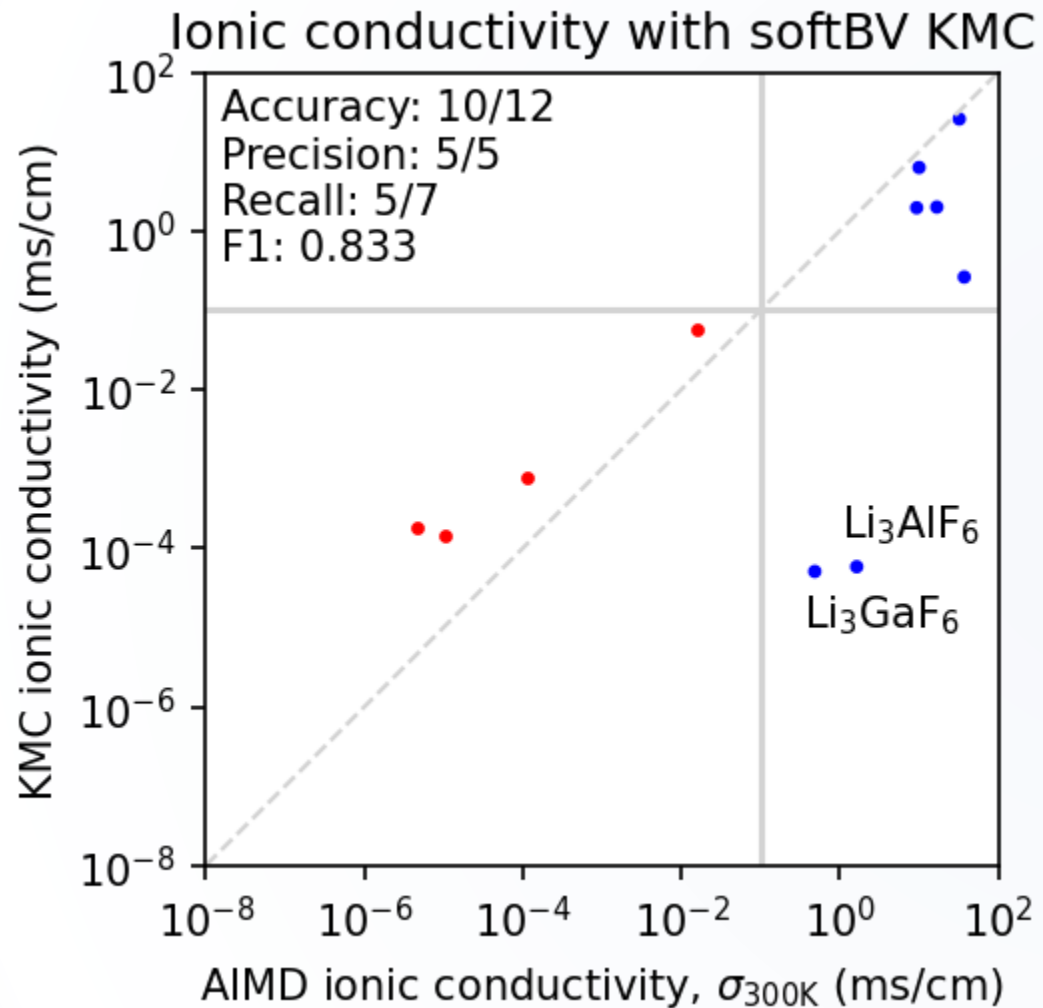
- AIMD reference includes few, high temperatures
- Transferability of softBV parametrization

Predicting activation energy



Both methods produce accurate predictions for activation energy

Predicting room temperature ionic conductivity



Precision

False Positive	True Pos.

Recall

	True Pos.
	False Neg.

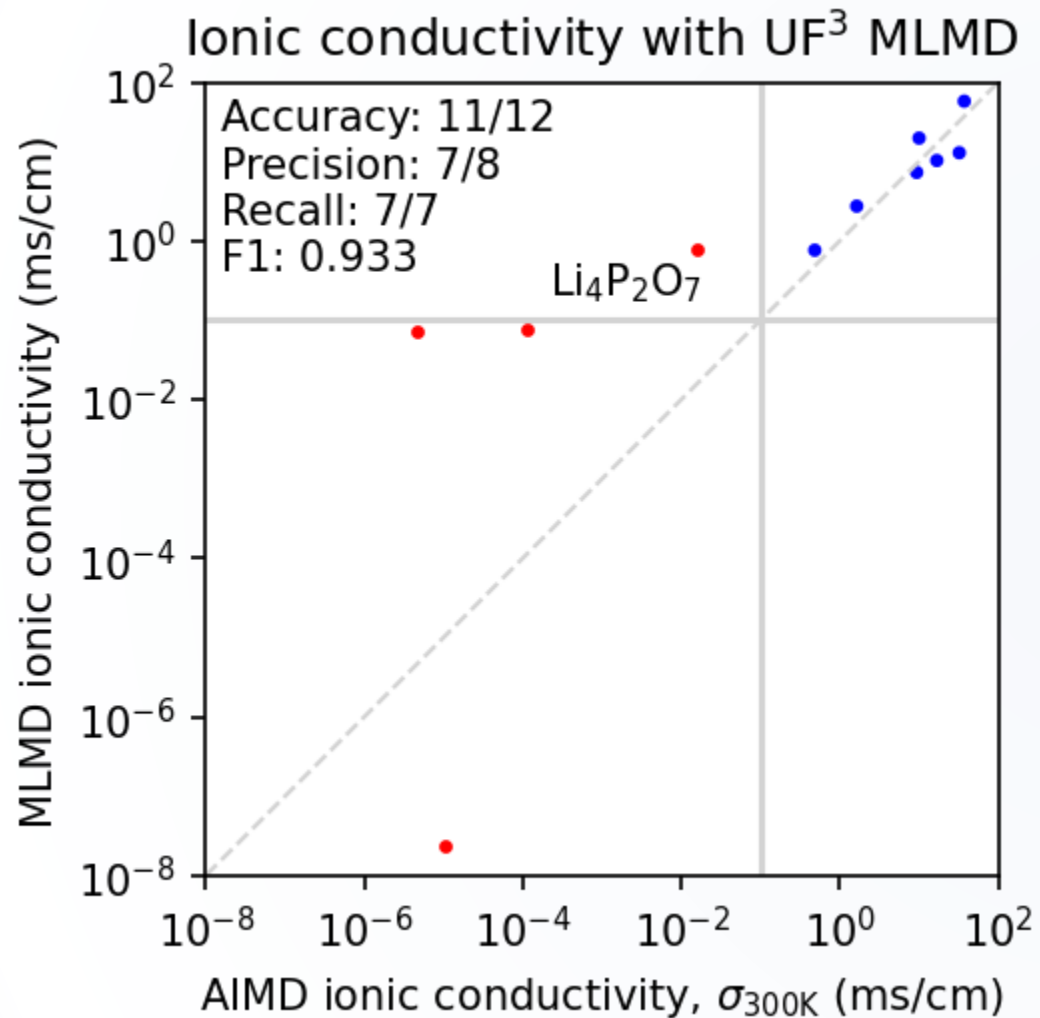
F1-score

False Positive	True Pos.
	False Neg.

Accuracy

False Positive	True Pos.
True Negative	False Neg.

Predicting room temperature ionic conductivity



Precision

False Positive	True Pos.

Recall

	True Pos.
	False Neg.

F1-score

False Positive	True Pos.
	False Neg.

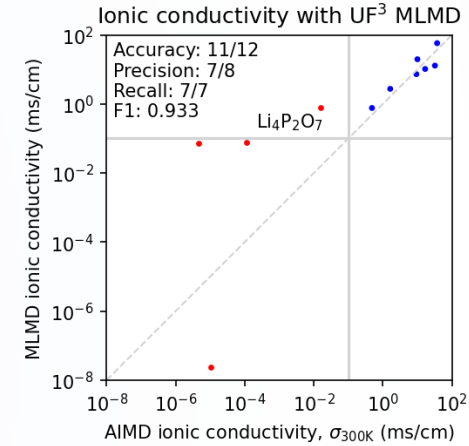
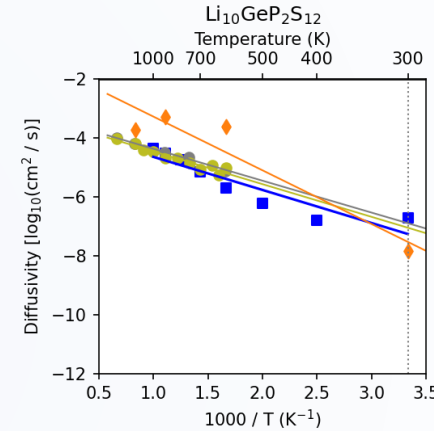
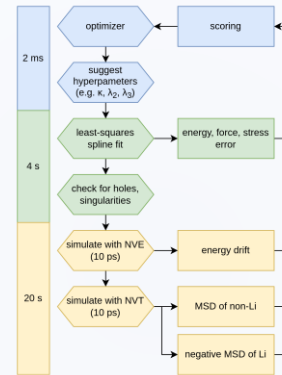
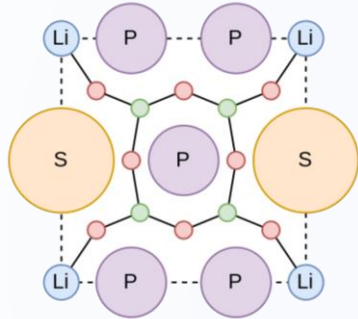
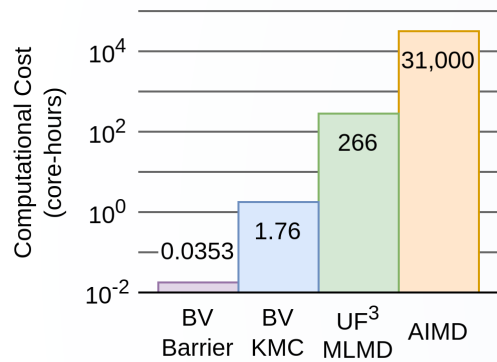
Accuracy

False Positive	True Pos.
True Negative	False Neg.

Hierarchical screening for Li-based solid electrolytes using fast, interpretable machine-learned potentials

Stephen R. Xie, Shreyas J. Honrao, John W. Lawson

Computational Materials Group, NASA Ames Research Center, Moffett Field, CA, USA



- Pipeline for high-throughput ionic conductivity predictions
- Validating bond valence methods for screening solid-state electrolytes
- Automatic fitting of ultra-fast, interpretable machine-learning force fields